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Recent Experiments on the Casimir Effect: Description and Analysis

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1 Motivations

After its prediction in 1948 [1], the Casimir force has been observed in a number of 'historic' experiments which confirmed its existence and main properties [2, 3, 4]. The Casimir force has recently been measured with a largely improved experimental precision [5] which should allow for an accurate comparison between measured values of the force and theoretical predictions. This comparison is interesting for various reasons.

The Casimir force is the most accessible effect of vacuum fluctuations in the macroscopic world. As the existence of vacuum energy raises difficulties at the interface between the theories of quantum and gravitational phenomena, it is worth testing this effect with the greatest care and highest accuracy [6]. But the comparison between theory and experiment should take into account the important differences between the real experimental conditions and the ideal situation considered by Casimir.

Casimir calculated the force between a pair of perfectly smooth, flat and parallel plates in the limit of zero temperature and perfect reflection. He found an expression for the force F_{Cas} and the corresponding energy E_{Cas} which only depend on the distance L, the area A and two fundamental constants, the speed of light c and Planck constant $\hbar$

$$\begin{aligned} F_{\mathrm{Cas}} &= \frac{\hbar c \pi^2 A}{240 L^4} = -\frac{\mathrm{d}E_{\mathrm{Cas}}}{\mathrm{d}L} \\ E_{\mathrm{Cas}} &= \frac{\hbar c \pi^2 A}{720 L^3} \end{aligned} \tag{1}$$

Each transverse dimension of the plates has been supposed to be much larger than L. Conventions of sign are chosen so that F_{Cas} and E_{Cas} are positive. They correspond to an attractive force ($\sim 0.1\mu$N for $A = 1$cm^2 and $L = 1\mu$m) and a binding energy.

Most experiments have been performed with a sphere-plane geometry which differs from the plane-plane geometry considered by Casimir. Since no exact result is available for the former geometry, the force is derived from the Deriagin approximation [7], often called in a somewhat improper manner the proximity force theorem. With this approximation, the force is obtained as the integral of force contributions corresponding to the various inter-plate distances as if these contributions were independent. In the plane-sphere geometry, the force is thus determined by the radius R of the sphere and by the Casimir energy as evaluated in the plane-plane configuration. As discussed in more detail below, the accuracy of this approximation is not really mastered. This

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difficulty also affects the accuracy in the evaluation of the surface roughness corrections which is again based on the Deriagin approximation.

The fact that the Casimir force (1) only depends on fundamental constants and geometrical features is remarkable. In particular it is independent of the fine structure constant which appears in the expression of the atomic Van der Waals forces. This universality property is related to the assumption of perfect reflection used by Casimir in his derivation. Perfect mirrors correspond to a saturated response to the fields since they reflect 100 % of the incoming light. This explains why the Casimir effect, though it has its microscopic origin in the interaction of electrons with electromagnetic fields, does not depend on the fine structure constant. Now, real mirrors are not perfect reflectors. The most precise experiments are performed with metallic mirrors which behave as nearly perfect reflectors at frequencies smaller than a characteristic plasma frequency but become poor reflectors at higher frequencies. Hence the Casimir expression has to be modified to account for the effect of finite conductivity. At the same time, experiments are performed at room temperature whereas the Casimir formula (1) only holds in vacuum, that is at zero temperature.

A precise knowledge of the Casimir force is a key point in many accurate force measurements for distances ranging from nanometer to millimeter. These experiments are motivated either by tests of Newtonian gravity at millimetric distances [8, 9] or by searches for new weak forces predicted in theoretical unification models with nanometric to millimetric ranges [10, 11, 12, 13, 14]. Basically, they aim at putting limits on deviations from present standard theory through a comparison of experimental results with theoretical expectations. The Casimir force is the dominant force between two neutral non-magnetic objects in the range of interest so that any new force would appear as a difference between experimental measurements and theoretical expectations of the Casimir force.

As far as the aim of a theory-experiment comparison is concerned, the accuracy of theory is as crucial as the precision of experiments. If a given accuracy, say at the 1% level, is aimed at in the comparison, then the theoretical as well as experimental accuracy have to be mastered at this level independently from each other. Since the various corrections to the Casimir formula which have already been alluded to may have a magnitude much larger than the 1% level, a high-accuracy comparison necessarily requires a precise analysis of the differences between the ideal case considered by Casimir and real situations studied in experiments.

2 Experiments before 1997

We first review some of the experiments performed before 1997.

The first experiment to measure the Casimir force between two metals was carried out by Spaarnay in 1958 [15]. A force balance based on a spring balance was used to measure the force between two flat neutral plates for distances between 0.5 and 2μm. Measurements were carried out for Al-Al, Cr-Cr and Cr-steel plates through electromechanical techniques. Spaarnay discussed the major difficulties of the experiments, in particular the control of the parallelism of the two plates, the determination of the distance between them, and the control of the neutrality of the two metal plates which is delicate since the Casimir force can easily be masked by electrostatic forces. The experiment gave evidence of an attractive force between the two plates and Sparnaay cautiously reported that "the observed attractions do not contradict Casimir's theoretical prediction". For the sake of comparison with recent experiments described below, an error bar of the order of 100 % may be attributed to this experiment.

Probably the first unambiguous measurement of the Casimir force between metallic surfaces was performed by van Blokland and Overbeek in 1978 [16]. The force was measured with the help of a spring balance between a lens and a flat plate, both coated with 50-100nm thick chromium layers, for distances from 132 to 670nm, measured by determining the capacitance of the system. The use of a lens instead of a second flat plate simplified the control of the geometry by suppressing the problem of parallelism. The force in this configuration was evaluated with the help of Deriagin's approximation discussed in more detail below since it also plays a key role in recent experiments. The investigators compared their experimental results to theoretical calculations using the Lifshitz theory for chromium and concluded to an agreement between the measured and calculated force values, confirming for the first time the effect of finite conductivity. One may estimate the accuracy to be of the order of 25%.

The Casimir force has been observed in a number of other experiments, in particular [17, 18, 19, 20]. More detailed or systematic reviews may be found in [2, 3, 4, 5].

3 Recent experiments

Recently new measurement techniques were used to measure the Casimir effect with improved accuracy. Quite a number of experiments have been carried out in the last years and we will describe some of them which seem to be the most significative ones.

In 1997 Steve Lamoreaux measured the Casimir force by using a torsion pendulum [21]. The force was measured between a metallized sphere and a flat metallic plate with controlled but unequal electrostatic potential. Since the electrostatic and Casimir forces were acting simultaneously, it was necessary to substract precisely the effect of the electrostatic force in order to deduce the value of the Casimir force. This measurement was made for distances between 0.6 and 6 microns. Comparison between the experimental results and the theoretical predictions was reported to be in agreement at the level of 5 %.

After the correction of inaccuracies in the initial report [22, 23, 24], the results of this experiment can be summarized as follows : the force has been measured, probably with an error bar of the order of 10 % at the shortest distances where the effect of finite conductivity of the Au and Cu metallic layers used in the experiments was unambiguously observed; the error bar was certainly much larger at distances larger than a few μm where the magnitude of the force is much weaker; this probably explains why the temperature correction has not been seen though it should have been seen at the largest distance $\sim 6\mu$m explored in the experiment (see below). It is difficult to be more affirmative on this topic, in particular because this experiment was stopped by the relocation of Steve Lamoreaux.

Shortly after this publication, a second measurement was reported by Umar Mohideen [25] followed by several reports with an improved precision [26, 27]. This experiment is based on the use of an atomic force microscope (AFM). A metallized sphere is fixed on the cantilever of the microscope and brought to the close vicinity of a flat metallic plate, at a distance between 0.1 and 0.9μm. Both surfaces are put at the same electrostatic potential and the Casimir force is measured by the deflection of a laser beam on the top of the cantilever, as shown on Figure 1.

The comparison between experimental results and theoretical predictions has been performed for Al and Au coated surfaces. A typical experimental accuracy at the level of 1% is obtained with a comparable agreement with theory, as depicted on Figure 2. Theoretical points are based on the methods described below. They take into account the effect of roughness. The same group has also studied the effect of sinusoidal corrugations on the properties of the Casimir force [28, 29].

An independent measurement has been published in 2000 by Thomas Ederth [30] who also used an AFM with the same working principle as for Mohideen's experiments. The force was measured between two neutral metallic crossed cylinders (curvature 10 mm) at short distances ranging from 20 to 100nm. Great efforts allowed Ederth to reduce the surface roughness down to a level of about 3nm, to be compared with the usual value of the order of 30nm which is due to the sputtering techniques used to coat the mirrors. After a careful error analysis, Ederth concluded to an accuracy at the level of several %.

We also want to mention experiments done by Federico Capasso and his group at Bell Labs,

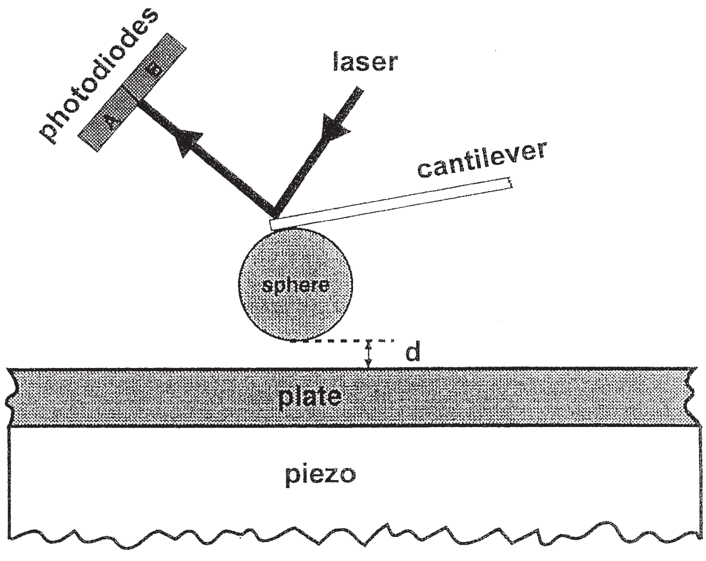

Figure 1: Experimental setup of the Casimir force measurement in [25, 26, 27]. The force is measured between the sphere and the plate with the distance of closest approach d (denoted L in the present report). The sphere is fixed on the cantilever of an AFM and its position measured by the deflection of a laser beam on the top of the cantilever. With kind courtesy of Umar Mohideen.

which observe the Casimir effect on microelectromechanical systems (MEMS) [31, 32]. The latter are movable structures fabricated on a semiconductor wafer through integrated circuit technology and they are used as a new generation of sensors and actuators working in the micrometer or submicrometer distance range. The experiment is again similar in its principle to those of the Mohideen group. The Casimir force is measured between a polystyrene sphere and a polysilicon plate with metallic coatings. The plate is suspended so that it could rotate around an axis. The variation of the plate rotation angle when the sphere is approached to a distance between 100nm and 1μm reveals the Casimir force with a magnitude agreeing with theory. When the plate is set into oscillation, frequency shifts, histeretic behavior and bistability are observed, again in agreement with the effect of the Casimir force predicted by the theory. The main interest of these experiments is to show that the Casimir force plays a significant role in systems of technological interest like the MEMS. This is not surprising since it is the dominant force in the micrometer range and this experiment shows that mechanical effects of quantum vacuum fluctuations have to be taken into account in micro- or nanotechnology [33].

Experiments described in the present section up to this point use a sphere-plane geometry or a crossed cylinders geometry. Their analysis relies on the accuracy of the Deriagin approximation which is not precisely known. This is not the case for the experiments performed in the initial Casimir geometry with two parallel flat plates. A measurement in this geometry has recently been reported on by Bressi, Carugno, Onofrio and Ruoso [34]. The force is observed between two parallel flat plates coated with chromium, one of which is mounted on a silicon cantilever while the other one is fixed on a rigid piezoelectric stack. The plate fixed on the piezoelectric stack is set into oscillatory motion and this induces a varying Casimir force onto the plate mounted on the cantilever. The motion of the latter is then monitored by using a tunneling electromechanical transducer. The measurement has been performed for distances between 0.5 and 3μm and the

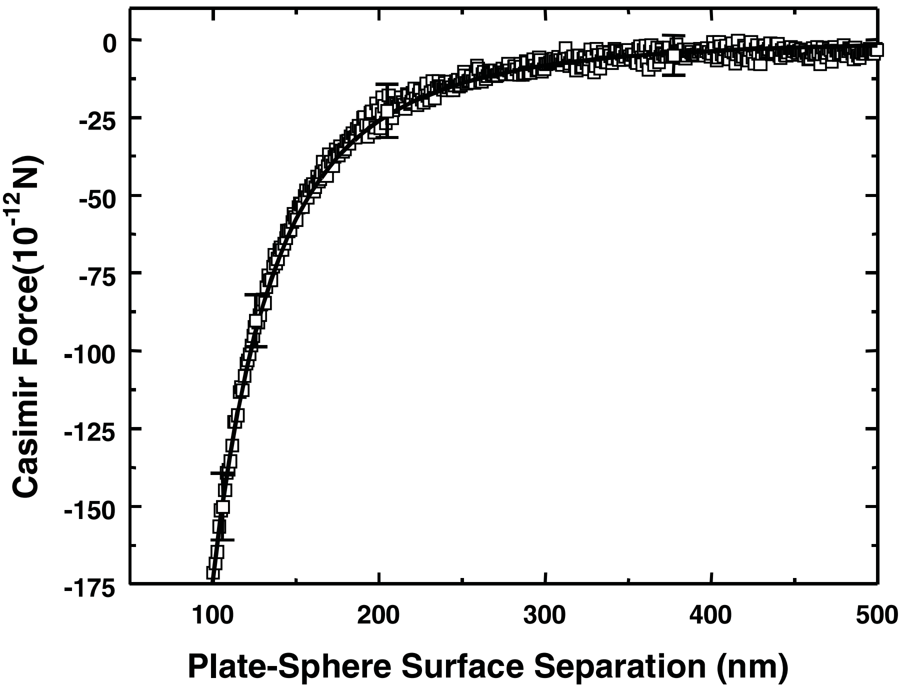

Figure 2: Comparison between experimentally measured values and theoretical predictions of the Casimir force, as reported in [26]; the squares and bars represent experimental points and errors bars for a few of them; the solid line represents theoretical predictions. With kind courtesy of Umar Mohideen.

result has been found to agree with theory at the 15% precision level.

4 The effect of imperfect reflection

As explained in the introduction, a precise theory-experiment comparison requires not only a detailed control of the experiments but also a careful estimation of the theoretical expectation of the force in the real conditions of the experiments. We begin here by the more spectacular "correction" to the ideal Casimir formula (1) which is associated with imperfect reflection of mirrors.

No real mirror can be considered as a perfect reflector at all field frequencies. In particular, the most precise experiments are performed with metallic mirrors which show perfect reflection only at frequencies smaller than a characteristic plasma frequency ω_{P} which depends on the the properties of conduction electrons in the metal. Hence the Casimir force between metal plates does fit the ideal Casimir formula (1) only at distances L much larger than the plasma wavelength

$$\lambda_{\mathrm{P}} = \frac{2\pi c}{\omega_{\mathrm{P}}} \tag{2}$$

For metals used in the recent experiments, this wavelength lies in the 0.1μm range (107nm for Al and 136nm for Cu and Au). At distances smaller or of the order of the plasma wavelength, the finite conductivity of the metal has a significant effect on the force. The idea has been known since a long time [35, 36] but the investigation of the effect of imperfect reflection has been systematically developed only recently.

We first consider the initial Casimir geometry with perfectly plane, flat and parallel plates at zero temperature. We thus restrict our attention on the effect of the reflection properties of the mirrors described by scattering amplitudes which depend on the frequency of the incoming field. Assuming that these amplitudes obey general properties of unitarity, high-frequency transparency and causality, one derives a regular expression of Casimir force which is free from the divergences usually associated with the infiniteness of vacuum energy. The cavity formed by the two mirrors can be dealt with by using the Fabry-Pérot theory. Vacuum field fluctuations impinging the cavity have their energy either enhanced or decreased inside the cavity, depending on whether their frequency is resonant or not with a cavity mode. The radiation pressure associated with these fluctuations exerts a force on the mirrors which is directed either inwards or outwards respectively. Finally, it is the balance between the inward and outward contributions, when they are integrated over the field frequencies and incidence angles, which gives the net Casimir force [37].

The techniques of analytical continuation of the response functions in the complex plane already used in [35] allow one to write the Casimir force as an integral over imaginary frequencies $\omega = i\xi$ with ξ real

$$\begin{aligned} F &= \frac{\hbar A}{\pi} \sum_p \int \frac{\mathrm{d}^2\mathbf{k}}{4\pi^2} \int_0^\infty \mathrm{d}\xi \frac{\kappa r_1^p\left[i\xi,\mathbf{k}\right] r_2^p\left[i\xi,\mathbf{k}\right]}{e^{2\kappa L} - r_1^p\left[i\xi,\mathbf{k}\right] r_2^p\left[i\xi,\mathbf{k}\right]} \\ & \kappa \equiv \sqrt{\mathbf{k}^2+\xi^2} \end{aligned} \tag{3}$$

$r_j^p\left[\omega,\mathbf{k}\right]$ is the reflection amplitude for the two mirrors $j = 1, 2$ and the field mode characterized by a frequency ω, a tranverse wavevector $\mathbf{k}$ (transverse means orthogonal to the main direction of the cavity, that is also parallel to the plane of the plates) and a polarization p. The amplitudes appear in the expression (3) at imaginary frequencies $\omega = i\xi$ where they have real and positive values. The fraction appearing in (3) represents the difference between the radiation pressures on outer and inner sides of the cavity after the continuation to the imaginary axis. It is determined by the product of the reflection amplitudes of the two mirrors and by an exponential factor $e^{2\kappa L}$ representing the propagation dephasing for the field after a roundtrip in the cavity, that is a propagation length $2L$. Expression (3) includes the contribution of the modes freely propagating inside and outside the cavity but also the contribution of evanescent waves confined to the vicinity of the mirrors.

Equation (3) is a convergent integral for any couple of mirrors described by scattering amplitudes obeying the properties of causality, passivity and high frequency transparency. This means that the potential divergence associated with the infiniteness of vacuum energy has been cured by using the physical properties of scattering amplitudes, that is also by describing mirrors just as opticians do describe mirrors. Furthermore expression (3) does not depend on any particular microscopic model but may be applied to any reflection amplitude obeying the general properties already discussed.

The ideal Casimir result is recovered at the limit where mirrors may be considered as perfect over the frequency range of interest, that is essentially over the first few resonance frequencies of the cavity [37]. Otherwise, the effect of imperfect reflection is described by a reduction factor η_F which multiplies the ideal Casimir expression (1) to give the force F between real mirrors

$$F = \eta_\mathrm{F} F_\mathrm{Cas} \tag{4}$$

In order to go further, we have to specialize the general expression (3) to a model of mirrors. The commonly used model corresponds to reflection on bulk mirrors with an optical response described by a dielectric function $\varepsilon\left(\omega\right)$. The reflection amplitudes corresponding to the two polarizations $p = \mathrm{TE}, \mathrm{TM}$ are thus given by the Fresnel formulas for each mirror

$$\begin{aligned} r_j^{\mathrm{TE}}[i\xi, \mathbf{k}] &= -\frac{\sqrt{\xi^2 \varepsilon(i\xi) + c^2\mathbf{k}^2} - c\kappa}{\sqrt{\xi^2 \varepsilon(i\xi) + c^2\mathbf{k}^2} + c\kappa} \\ r_j^{\mathrm{TM}}[i\xi, \mathbf{k}] &= \frac{\sqrt{\xi^2 \varepsilon(i\xi) + c^2\mathbf{k}^2} - c\kappa\varepsilon(i\xi)}{\sqrt{\xi^2 \varepsilon(i\xi) + c^2\mathbf{k}^2} + c\kappa\varepsilon(i\xi)} \end{aligned} \tag{5}$$

Taken together, relations (3,5) reproduce the Lifshitz expression for the Casimir force [35]. It is worth stressing however that relations (3) have a wider domain of validity since, as already discussed, they allow one to deal with more general scattering amplitudes than (5).

The optical response of conduction electrons in metals is approximately described by a plasma model, that is by a dielectric function

$$\varepsilon(\omega) = 1 - \frac{\omega_{\mathrm{P}}^2}{\omega^2} \tag{6}$$

A better description is given by the Drude model which accounts for the relaxation of conduction electrons

$$\varepsilon(\omega) = 1 - \frac{\omega_{\mathrm{P}}^2}{\omega(\omega + i\gamma)} \tag{7}$$

Since the ratio $\frac{\gamma}{\omega_{\mathrm{P}}}$ is much smaller than unity, the relaxation parameter γ has a significant effect on ε only at frequencies where the latter is much larger than unity and where, accordingly, the mirror is nearly perfectly reflecting. It follows that relaxation has a small influence on the value of the Casimir force [38].

In contrast, the modification of the dielectric constant due to interband transitions has an observable effect on the Casimir force measured at distances of the order of the plasma wavelength [38]. This appears on the results of numerically integrated values of the reduction factor η_{F} shown on Figure (3). The solid line represents the factor calculated for two identical Au mirrors described by the plasma model with the plasma wavelength $\lambda_{\mathrm{P}} = 136\mathrm{nm}$ corresponding to Au. Meanwhile, the dashed line represents the factor calculated by using the tabulated optical data for Au [38].

This figure clearly shows that the effect of imperfect reflection is important at the smallest distances explored in the experiments : the reduction factor is of the order of 50% for Au mirrors at a distance around $0.1\mu\mathrm{m}$. It also appears that a careful description of the optical properties of metals is necessary to obtain a precise estimation of the force : in particular, the description of metals by the plasma model is not sufficient if an accuracy in the 1% range is aimed at.

5 The effect of temperature

The preceding estimations were corresponding to experiments which would be performed at zero temperature. But all experiments to date have been performed at room temperature and the radiation pressure of thermal field fluctuations has a significant contribution to the force at distances larger than or of the order of a thermal wavelength [39, 40]

$$\lambda_{\mathrm{T}} = \frac{\hbar c}{k_{\mathrm{B}} T} \tag{8}$$

with $\lambda_{\mathrm{T}} \sim 7\mu\mathrm{m}$ at room temperature.

It is in principle quite simple to describe the effect of thermal field fluctuations which are superimposed to vacuum fluctuations. At zero temperature indeed, the field energy per mode is

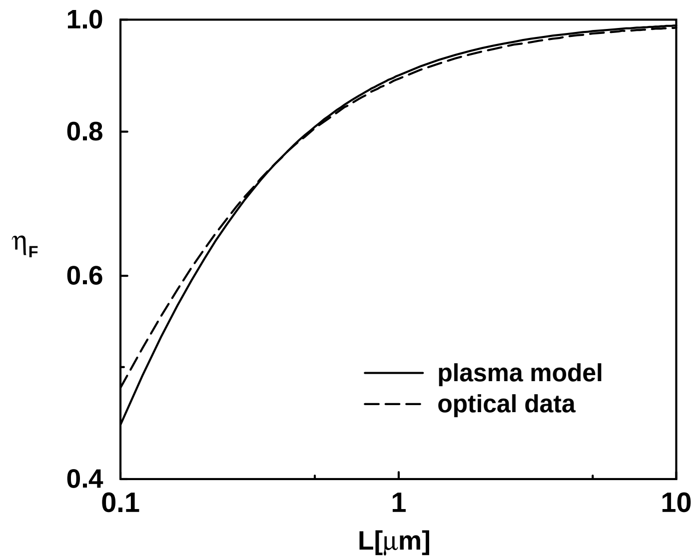

Figure 3: Reduction factor η_{F} for the Casimir force between two identical Au mirrors at zero temperature as a function of the distance L. The solid and dashed lines correspond to evaluations based respectively on the plasma model with $\lambda_{\mathrm{P}} = 136\mathrm{nm}$ and on tabulated optical data for Au.

simply the vacuum contribution $\frac{1}{2}\hbar\omega$. At a non zero temperature, the field energy is the sum of this vacuum contribution and of the energy of the mean number n of photons per mode given by Planck law

$$\frac{1}{2}\hbar\omega \quad \longrightarrow \quad \left(\frac{1}{2} + n\right)\hbar\omega \tag{9}$$

This means that the contribution of a mode of frequency ω to the Casimir force has to be multiplied by a factor

$$1 + 2n\left(\omega\right) \quad = \quad \frac{1}{\tanh \frac{\hbar\omega}{2k_{\mathrm{B}}T}} \tag{10}$$

After the analytical continuation to the imaginary axis, expression (3) has to be modified by inserting a factor $1 + 2n\left(i\xi\right)$ in the integrand. This factor has at frequencies $\xi_m = m\frac{2\pi k_{\mathrm{B}}T}{\hbar}$ (m integer) which must be treated with great care. It may be expanded as a series of exponential functions, which leads to equation (7) of [41] used in this paper as the starting point of numerical integration of the correction factor η_{F}.

This correction factor is drawn on Figure 4 as a function of the distance L. Here, we have chosen to consider two identical Al mirrors described by a plasma model with the plasma wavelength $\lambda_{\mathrm{P}} = 107\mathrm{nm}$. The solid line represents the correction factor η_{F} in such a configuration at room temperature $T = 300\mathrm{K}$. For the sake of comparison, we have also represented, as the dashed line, the plasma correction $\eta_{\mathrm{F}}^{\mathrm{P}}$ evaluated with the same mirrors at zero temperature and, as the dotted-dashed line, the thermal correction $\eta_{\mathrm{F}}^{\mathrm{T}}$ evaluated with perfect reflectors at room temperature.

The plasma correction factor $\eta_{\mathrm{F}}^{\mathrm{P}}$ describes only the effect of imperfect reflection and corresponds to the reduction of the force discussed in the preceding section. Meanwhile the thermal

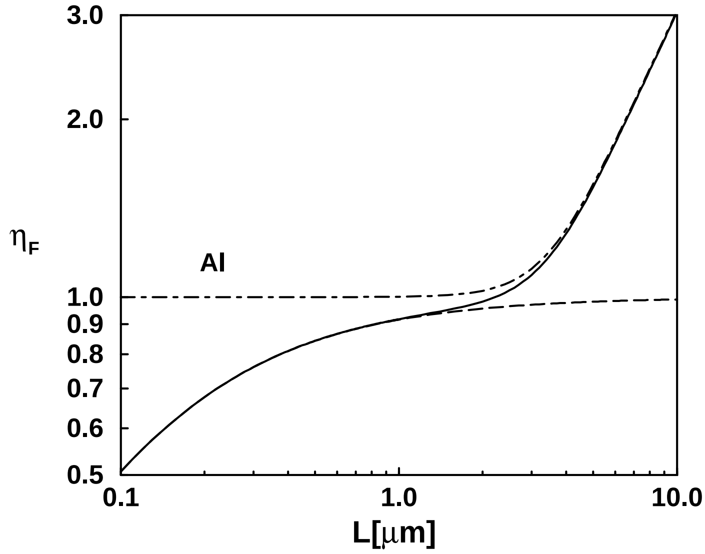

Figure 4: Correction factors for the Casimir force between two identical Al mirrors described by a plasma model with $\lambda_P = 107$nm) at room temperature $T = 300$K as functions of the distance L. The solid, dashed and dotted-dashed lines represent respectively the whole correction factor η_F, the plasma correction factor η_F^P describing only the effect of imperfect reflection and the thermal correction factor η_F^T describing only the effect of temperature.

correction factor η_F^T describes only the effect of temperature : it is computed for perfect reflection and corresponds to an increase of the force. The two factors are appreciable respectively at distances smaller than 1μm and larger than 1μm. It follows that the whole correction η_F giving the force F when both effects are simultaneously accounted for is essentially equal to the product of the plasma and thermal correction factors. This is however an approximation the accuracy of which has to be carefully discussed when a precise evaluation is aimed at.

In order to evaluate the quality of this approximation, it is worth writing the whole correction factor as

$$\eta_F = \eta_F^P \eta_F^T (1 + \delta_F) \tag{11}$$

A null value for δ_F would mean that the whole correction factor may effectively be evaluated as the product of the plasma and thermal corrections computed independently from each other. In contrast, a non null value represents a correlation of the plasma and thermal corrections.

The variation of the correlation factor δ_F has been discussed in a detailed manner in [41, 42]. It turns out that this correlation should be taken into account when an accuracy at or beyond the 1% level is needed. This stems from the fact that the correlation scales as the ratio $\frac{\lambda_P}{\lambda_T}$ of the two wavelengths which characterize respectively the plasma and thermal effects and is of the order of 10^{-2} for ordinary metals at room temperature. The correlation factor is appreciable at distances larger than 1μm where the plasma model is known to be a good effective description of the metallic optical response. This justifies the use of this model in [41, 42]. At short distances, say around 0.1-0.5μm, a more complete description of the metallic optical response is needed but the temperature correction is negligible in this distance range. Note also that an analytical approximation of the correlation factor has been given in [41] through a perturbative development of the force to first order in $\frac{\lambda_P}{\lambda_T}$. The resulting expression is found to fit well the results of the complete numerical integration, with an accuracy much better that the 1% level. It provides one with a simple method for getting an accurate theoretical expectation of the Casimir force throughout the whole distance range explored in the experiments.

For the sake of completeness, we may mention that the evaluation of the Casimir force between real mirrors at a non zero temperature has recently given rise to a burst of controversial results [43, 44, 45] (see also [46, 47, 48, 49, 50]). As far as this controversy is concerned, the evaluations deduced here from [41] are in agreement with the results of [45] and at variance with the conclusions of [43, 44].

6 Effect of the geometry

It now remains to describe how the effect of geometry is included in the theoretical estimations of the Casimir force.

As already discussed, most experiments are performed in a sphere-plane geometry which differs from the plane-plane geometry for which exact expressions are available. The force in the former geometry is derived from the Deriagin approximation [7] which basically amounts to sum up the contributions corresponding to various inter-plate distances as if these contributions were independent. In the plane-sphere geometry, the result is simply determined by the radius R of the sphere and by the Casimir energy as evaluated in the plane-plane configuration

$$\begin{aligned} F_{\mathrm{sphere-plane}} &= \frac{2\pi R}{A} E_{\mathrm{plane-plane}} \\ E_{\mathrm{plane-plane}} &= \int_L^\infty \mathrm{d}x \; F_{\mathrm{plane-plane}}(x) = \eta_\mathrm{E} E_\mathrm{Cas} \end{aligned} \tag{12}$$

We have introduced a correction factor η_E for the Casimir energy, evaluated for the plane-plane geometry in the same manner as η_F for the Casimir force in (4).

Collecting these results leads to the final expression of the Casimir force in the sphere-plane geometry

$$F_{\mathrm{sphere-plane}} = \frac{\hbar c \pi^3 R}{360 L^3} \eta_\mathrm{E} \tag{13}$$

We have shown on Figure (5) the numerically integrated values of the reduction factor η_E for two identical Au mirrors at zero temperature. As on Figure (3), the solid line represents the factor calculated for mirrors described by the plasma model with $\lambda_\mathrm{P} = 136$nm whereas the dashed line represents the factor deduced from the tabulated optical data for Au [38].

We have considered here the case of a null temperature so that the evaluation is correct only at distances smaller than 1μm which corresponds to the most precise results obtained by Mohideen group. At longer distances, the temperature correction has to be taken into account and this can be done by following the method presented in the preceding section.

At short distances, surface roughness corrections are also significant. They are included by using again the Deriagin approximation [51], which amounts to average the value of the Casimir forces on the various values of the inter-plate distances. A recent publication [52] opens the route to more precise evaluations of the plate corrugation and, potentially, of the surface roughness. As it could be expected, the effect of corrugation is found to depend on the wavelength of the surface perturbation and not only on its amplitude. In this new evaluation, the result of the Deriagin approximation is recovered only at the limit of large wavelengths or, equivalently, small wavevectors of the surface perturbation.

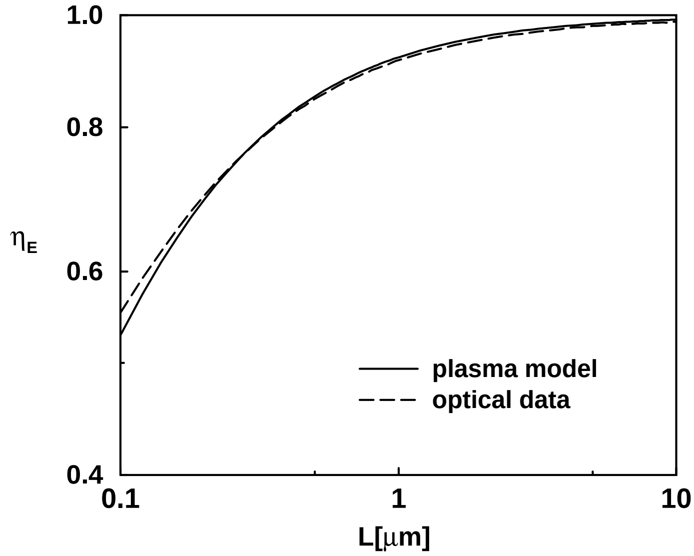

Figure 5: Reduction factor η_{E} for the Casimir energy between two identical Au mirrors at zero temperature as a function of the distance L; same conventions as on figure 3.

At this point, it is worth noting that the problem is in fact a more general deficiency of the Deriagin approximation. This approximation amounts to add the contributions corresponding to different distances but we know with certainty that the Casimir force is not additive (see a recent detailed discussion in [53]). As a result, the Deriagin method, often improperly called the proximity force theorem, can not be exact. And, what is more problematic when addressing the problem of accuracy of theoretical predictions, we do not know in general the accuracy of this approximation.

Summary

We will now sum up the results obtained so far in experimental measurements and comparison with theoretical expectations of the Casimir force.

It is clear that the Casimir effect has now been unambiguously observed : the experimental precision is already at the 1% level and it will certainly be improved in the future. This precision has allowed the experiments to observe the effect of imperfect reflection. However, the effect of temperature has not been seen at the largest distances explored in the experiments although it should have been. This is probably due to an insufficient precision in these experiments.

An accurate theory-experiment comparison requires not only precise measurements but also accurate and reliable theoretical estimations. Important advances have been recently reported for the estimation of the effects of imperfect reflection and non null temperature. Efforts are presently focussed on the effects of geometry and surface roughness. It is worth keeping in mind that not only the accuracy of the approximations used to treat these effects should be carefully studied for perfect mirrors in vacuum but also that the corrections due to these effects are probably correlated to the effects of imperfect reflection and temperature in the same manner as the two latter effects are now known to be correlated to each other.

An attractive alternative is to come back to the initial plane-plane geometry but experiments in this geometry have not been able so far to reach the precision of sphere-plane experiments.

New advances are expected to occur quite soon in this domain, both on the experimental and theoretical sides. These new results will probably allow one to progress towards an improvement of the precision of the theory-experiment comparison. Any such improvement, at the 1% level or beyond, is important, since it either confirms a central prediction of Quantum Field Theory or otherwise reveals surprising new results in the domain of forces with nanometric to millimetric ranges.

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