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Tunneling Experiments in the Fractional Quantum Hall Effect Regime

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Abstract. The Fractional Quantum Hall effect provides a unique example of a quantum system with fractional quantum numbers. We review the tunneling experiments which have brought into evidence the fractionally charged excitations, the fractional occupation of the quantum states and the non-linear quantum transport related to the chiral Luttinger liquids properties.

1 Introduction

The quantum Hall effect is one of the most remarkable macroscopic manifestation of quantum mechanics in condensed matter after superconductivity and superfluidity. The phenomenon is observed in a two-dimensional electrons gas (2DEG) at low temperature in a high perpendicular magnetic field. Landau Levels (LL) form due to cyclotron motion quantization in 2D and are highly degenerate. However, the degeneracy can be lifted by the interactions. The system can be viewed as a flat macroscopic atom made of 10^9 electrons. As for atoms or nuclei, particular values of the filling of the electronic states lead to more stable ground states with large *energy gap* for the excitations. The equivalent of magic atomic quantum numbers are integer or fractional values of the filling factor $\nu = p/q$ of the electronic quantum states (p and q integers). The filling factor, is given by the ratio of the electron density n_s to the density $n_\phi = B/\phi_0$ of flux quantum $\phi_0 = h/e$. It can be varied either by sweeping the magnetic field or by changing the electron density. The magic values of ν are experimentally revealed by plateaus in the Hall resistance $\frac{q}{p}\frac{h}{e^2}$. The Integer Quantum Hall Effect, discovered by Klitzing [1], occurs when $\nu = p$ ($q = 1$) when there is a complete filling of the degenerate LLs. The Fractional Quantum Hall Effect [2, 3] occurs at $\nu = p/q$ ($q = 2s + 1$, s integer). The underlying physics is the Coulomb interaction which lifts the LL degeneracy to form new correlated quantum liquids with energy gap and with topological excitations having a fractional charge $e/(2s + 1)$.

The Quantum Hall effect, and in particular the Fractional Quantum Hall effect [4], have completely renewed our knowledge of quantum excitations. Topological fractionally charged excitations [3], with anyonic or exclusonic fractional quantum statistics [5], composite fermions [6] or composite bosons [7], skyrmions [8, 9], etc., ... are the natural elementary excitations required to understand the quantum Hall effect. The quantum Hall effect have made real some concepts invented for the purpose of particle physics theories or used in mathematical physics for quantum integrable systems [10]. It is remarkable that Coulomb interaction and Fermi statistics, the simplest ingredients one can imagine, are responsible for a so rich physics. No interaction with the host material is needed as in the case of superconductivity. For macroscopic samples, however, a little amount of disorder is required to localize the topological excitations and thus to allow observation of Hall resistance quantization over a finite range of magnetic field (the Hall plateaus). It is a rare example where imperfections help to reveal a fundamental quantum effect. For narrow mesoscopic samples, disorder is to be avoid, and the QHE is revealed by the integer or fractional conductance quantization associated with the formation of chiral one dimensional edge modes. The properties of chiral edge modes are deeply related to the bulk properties of the Quantum Hall electron fluid.

I will focus here on tunneling experiments which allow for probing the fractional excitations and the fractional filling of the states. More general reviews on the quantum Hall effect can be found for example in Refs [11, 12, 9]. In these tunneling experiments, the charge transfer occurs between gapless chiral modes, called edge channels, which form at the periphery of a QHE fluid. Indeed, as the longitudinal conductance in the bulk vanishes, only these modes can generate a current in response to a potential drop. The edge channels can be easily connected to metallic contacts arranged at the periphery of the sample and then to an external circuit. In the IQHE they can be considered as good realization of 1D metals with the remarkable property that backscattering is suppressed by chirality. In the FQHE regime, they inherit from the bulk several non-trivial properties. First, they are no longer Fermi liquids. Their quantum dynamics is very similar to that of Tomonaga-Luttinger liquids predicted for 1D interacting electrons. The relevant excitations which propagate the charge information is no longer the fermionic Landau quasiparticle (the screened electron) but bosonic collective neutral modes (plasmons) instead. A remarkable consequence is the power law vanishing tunneling density of state (TDOS) at the Fermi energy. Indeed, an electron locally injected from an external contact into an FQH edge must excite many of these collective modes. If injected at the Fermi sea, no mode can be excited, and the tunneling rate vanishes (orthogonality catastrophy). A second non-trivial property inherited from the bulk is the possibility to extract from the edge a fractionally charged $e/(2s+1)$ quasiparticle for $\nu = p/(2s+1)$. Such quasiparticle tunneling between fractional edges is only observable when tunneling *through* the bulk FQHE liquid. An additional requirement due to the Luttinger liquid physics is a large bias voltage applied between the edges. Otherwise, at low voltage (and low temperature) near equilibrium, only integer charge is observed in experiments (the Luttinger liquid properties of the edges forces the quasiparticles to 'bunch' to form ordinary electrons). The detection of the fractionally charged quasiparticles have been made possible by the current noise generally associated with tunneling and called shot noise. This became recently possible thanks to the development of very sensitive current noise measurements in mesoscopic physics. For weak tunnel current, the temporal statistics of charge transfer is Poissonian, and the current noise is a direct measure of the charge carrier. The fractional quantum Hall effect is the first, and until now unique, example of a system with fractionally charged carriers. Also using shot noise, it has been also possible to follow the cross over from fractional to integer charges when reducing the bias voltage. At equilibrium, voltage lower than $k_B T$ the resonant tunneling of electrons between edge states can be used to probe the fraction of charge associated with the addition of a single flux quantum in the ground state using conductance measurement. The charge accumulated on a micrometer edge state ring is shown to vary by fractional increments with flux either by changing the magnetic field or by varying the size of the ring with a gate. This demonstrates that individual quantum sates participating to the formation of the collective ground state are actually filled by a fraction of electron. This is this fractional occupation which is responsible for the exact fractional quantization of the conductance.

The notes are organized as follows. Section II will describe the chiral edge one-dimensional modes which form in a finite 2D electron gas in perpendicular magnetic field. In section III, the chiral Luttinger liquid physics is presented. Tunneling experiments revealing the anomalous power law density of states will be reviewed. In section IV, non equilibrium experiments probing the charge carrier by measuring the shot noise of the current will be described. In V we will show how equilibrium resonant tunneling experiments can probe the fraction of charge which fills individual states participating to the ground state.

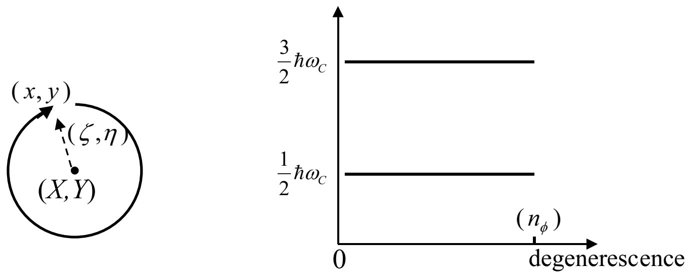

Figure 1: Left: decomposition of the electron coordinates (x, y) into cyclotron orbit coordinates (ξ, η) and the coordinates of the obit center (X, Y). Right: energy Landau levels formed by quantization of the cyclotron orbits

2 Tunneling in the Quantum Hall regime

2.1 Edge states in the integer quantum Hall regime

The kinetic energy $K = \frac{(\boldsymbol{p}-q\boldsymbol{A})^2}{2m^*}$, $q = -e$, of an electron moving freely in the plane perpendicular to a magnetic field $\mathbf{B} = B\widehat{\mathbf{z}}$ is quantized into Landau levels:

$$E_n = (n + \frac{1}{2})\hbar\omega_c \tag{1}$$

($\omega_c = eB/m^*$: cyclotron pulsation). This reflects the quantization of the cyclotron motion. As the energy depends on a single quantum number n while there are two degrees of freedom, there is a high degeneracy. The degeneracy comes from the freedom to choose the center of cyclotron orbits and is equal to the number $N_\Phi = n_\Phi S = eBS/h$ of magnetic flux quanta $\Phi_0 = h/e$ in the plane. To see this, one can replace the conjugate pairs of electron coordinates $[x, p_x]$ and $[y, p_y]$ by a new set of conjugate pairs, see Fig.1, using the cylindrical gauge $\mathbf{A} = (-By/2, Bx/2, 0)$:

$$[\xi, \eta] = [v_y/\omega_c, -v_x/\omega_c] = -i\hbar/eB \tag{2}$$

$$[X, Y] = i\hbar/eB \tag{3}$$

with:

$$(x, y) = (X + \xi, Y + \eta) \tag{4}$$

The Hamiltonian now writes $H = \frac{1}{2}m\omega_c^2(\xi^2 + \eta^2)$, so the first pair of conjugate coordinates represents the fast cyclotron motion. For an eigenstate $|n\rangle$ of H, the cyclotron radius is:

$$r_n = \langle n|\, \xi^2 + \eta^2\, |n\rangle^{1/2} = (n + \frac{1}{2})^{1/2} l_c \tag{5}$$

It increases with the orbital Landau level index n. The characteristic length $l_c = (\hbar/eB)^{1/2}$ is called the magnetic length. H does not depend on the second pair of coordinates $\mathbf{R} = (X, Y)$, the center around which electrons perform cyclotron orbits. However, orbit center positions can not be chosen completely freely in the plane as announced above. The commutation relation $[X, Y] = i\hbar/eB$ put restrictions on the number of possible distinct states. There is a finite degeneracy which is easy to estimate using the following analogy. The plane is similar to the semi-classical phase space (P, Q) of a one-dimensional system for which it is known that the effective area occupied by a quantum state is h as $[Q, P] = i\hbar$. Similarly, the area occupied by a quantum Hall state is h/eB, the area of a flux quantum. The degeneracy per unit area is thus n_Φ, the density of flux quanta; it is the same for all Landau levels. The semiclassical analogy will be usefull to get intuition about the *one-dimensional* character of the dynamics of electrons in *two dimensions* contrained to stay in a given Landau Level.

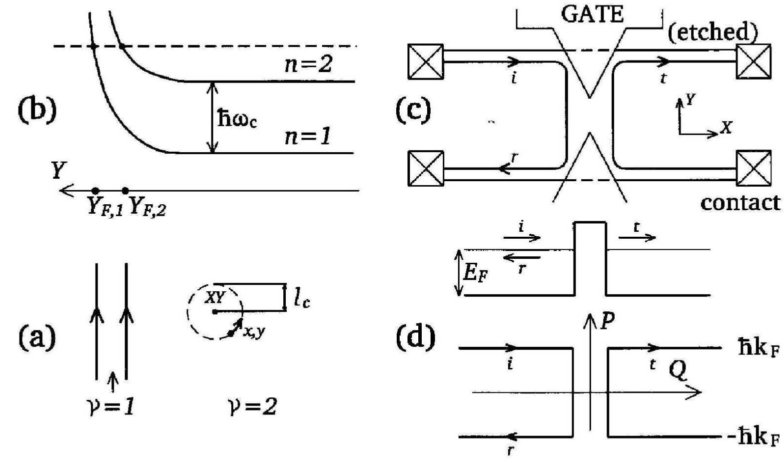

Figure 2: Left: (a) Schematic representation of edge states and of (b) the Landau level bending. (c) reflection of an edge state by a controlled artificial impurity called Quantum Point Contact. (d) analogy with 1D semiclassical trajectories in the phase space.

2.1.1 Edge states

In real sample of finite size, the 2DEG is bounded thanks to a permanent electric field directed perpendicular and toward the perimeter. The confining electric field compensates the long range Coulomb electronic repulsion and prevents electrons to escape from the area. It can be provided by ionized donor atoms in the host semiconductor lattice and uniformly distributed inside the region filled by electrons. Alternatively, an extra confinement can be provided by a gate negatively polarized with respect to the electrons and placed close and outside the electron area.

The Hamiltonian in presence of a potential $U(x, y)$:

$$H = \frac{(\boldsymbol{p} + e\boldsymbol{A})^2}{2m^*} + U(x, y) \tag{6}$$

can be simplified if the potential is smooth over the length l_c and $l_c |\nabla U| \ll \hbar\omega_c$. The mixing between Landau Levels can be neglected and $H \simeq \sum_n |n\rangle H_n \langle n|$. The dynamics of electrons within the n^{th} Landau level is described by the projected Hamiltonian:

$$H_n = (n + 1/2)\hbar\omega_c + U^{(n)}(X, Y) \tag{7}$$

where $U^{(n)}(X, Y) = \langle n| U(x, y) |n\rangle \simeq U(X, Y)$ is the confining potential averaged over the fast cyclotron motion. If the electric field due to confinement is along the $\widehat{y}$ direction, electrons drift along the boundary, the $\widehat{x}$ direction, with velocity $\frac{dX}{dt} = (1/eB)\partial U/\partial Y$. The Lorentz force compensates the electrostatic field, a direct consequence of the quantization of the velocity modulus ($\frac{d}{dt}(\boldsymbol{p} - (-e)\boldsymbol{A})^2 = 0$). In the bulk, $U \simeq 0$, the drift velocity is zero. Electrons do not move on average although performing fast cyclotron motion.

2.1.2 Edge channels

At zero temperature, electrons fill the Landau level up to a Fermi Energy E_F. The Fermi energy, here measured from the zero of kinetic energy, is defined by the exchange of electrons with a reservoir (practically: a contact somewhere on the edges). Here we disregard spin for simplicity and assume U translationally invariant along $\widehat{x}$ and vanishing in the bulk.

In the bulk, when $(p - \frac{1}{2})\hbar\omega_c < E_F < (p + \frac{1}{2})\hbar\omega_c$ electrons fill all the bulk states of the first p (integer) Landau Levels according to Fermi statistics. The filling factor is $\nu = p$. There is a gap

$\hbar\omega_c$ for creating internal excitations which leads to vanishing longitudinal conduction in the bulk. There is also an energy cost $E_F - (n + 1/2)\hbar\omega_c$ to extract an electron from the n^{th} Landau Level to the Fermi energy.

Toward edges, Landau levels are adiabatically bend by the potential $U(Y)$ and depopulate when crossing the Fermi energy, see Fig.2(a) and (b). For the n^{th} LL this occurs at $Y = Y_{F,n}$, when the energy cost $E_F - (n + 1/2)\hbar\omega_c - U(Y_{F,n})$ vanishes. This defines p lines along the edge with gapless excitations. This lines of gapless excitations restore conduction.

The one dimensional chiral conduction modes so formed are called edge channels. They can be connected to external contacts fixing their Fermi energy. The drift of electrons along the equipotential lines generated by the confining potential gives rise to a *persistent chiral current* . When the Fermi energy rises from E_F to $E_F + eV$, each mode contribute to increase the persistent chiral current by an equal contribution: $\Delta I = e \int_{Y_{F,n}(E_F)}^{Y_{F,n}(E_F+eV)} dy.n_\Phi(1/eB)\partial U/\partial Y = \frac{e^2}{h}V$. Thus each edge channel is associated with a conductance equal to the quantum of conductance $\frac{e^2}{h}$. This result can be equally viewed as a special case of the Landauer formula (which is valid in the more general case of quantum conductors, even in zero magnetic field) or as the quantization of Hall conductance. Landauer formula and quantized Hall conductance are direct consequence of the Pauli principle: the filling factor of quantum states is one.

2.1.3 Tunneling between edge channels

Edge channels are ideal one dimensional (chiral) conductors : the physical separation between pairs of opposite edge channels prevents backscattering and electrons propagate elastically over huge distances (∼mm at low temperature) as phonon scattering is reduced. These properties have made them a convenient tool to test the generalization of the Landauer formula: the Landauer-Büttiker relations [13, 14] derived in the context of the mesoscopic quantum transport. In order to do that it is necessary to induce intentionally elastic backscattering in a controllable way.

The tool used is a Quantum Point Contact (QPC) as shown in Fig.2(c). A negative potential applied on a metallic gate evaporated on top of the sample depletes electrons to realize a narrow constriction in the 2DEG. This allows a controllable modification of the boundaries of the sample. The separation between opposite pairs of edges channels of a given Landau level can be made so small that the overlap between wavefunctions lead to backscattering from one edge to the other. The QPC creates a saddle shape potential. When the potential at the saddle point is close but below the value $E_F - (n + \frac{1}{2})\hbar\omega_c$, electrons emitted from the upper left edge channel start to be reflected into the lower edge channel with probability $R \ll 1$ while they are still mostly transmitted with probability $T = 1 - R$. When the saddle point potential is above $U_{F,n}$ electrons are mostly reflected and rarely transmitted $T \ll 1$ and the reflection quickly reaches $R \precsim 1$.

For getting better intuition on edge channel tunneling, Fig.2(d) shows the semi classical analogy between the real space coordinates (Y, X) of the 2D Hall conductor and the (P, Q) phase space coordinate of a real 1D conductor. The physics of tunneling between opposite edge channels is clearly equivalent to that of the tunneling in a 1D system. However the chirality allows us to inject or detect electrons *at the four corners of the phase space*, something impossible with 1D systems.

Measuring the conductance is a good tool to know how many edge channels are transmitted. According to the Landauer formula, the conductance G is defined as the ratio of the current I through the QPC to the voltage difference V between the upper left and lower right contacts.

$$G = \frac{e^2}{h}(p - 1 + T) \tag{8}$$

if there are $p - 1$ channels transmitted while the p^{th} channel is partially transmitted with transmission T.

Figure 3 shows the reflection of edge channels starting from $\nu = 8$ in the bulk. One starts with 8 channels transmitted and when applying negative voltage on the gate the successive reflection 6 edge channels is observed by quantized plateaux in the resistance (here, the "access" resistance $h/8e^2$ has been subtracted).

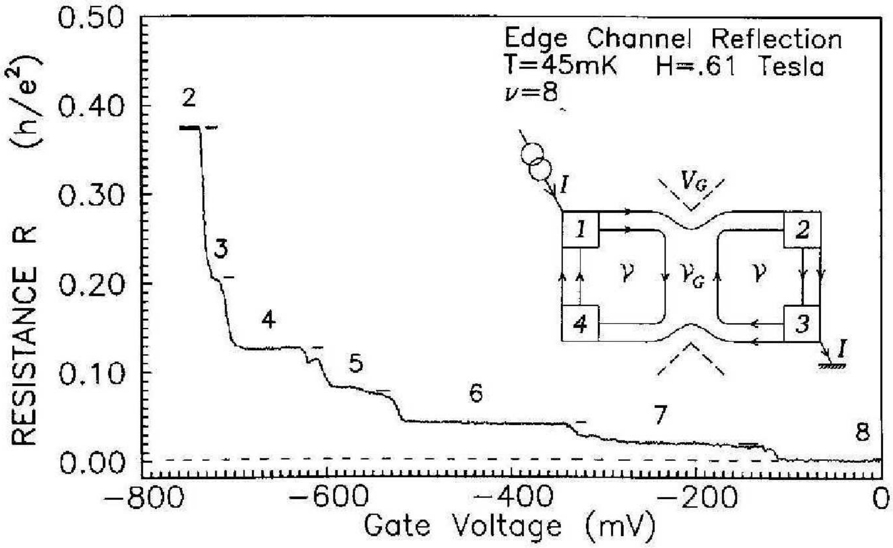

Figure 3: resistance of the QPC versus the QPC gate voltage showing the reflection of the six first edge states for $\nu = 8$, at T=45mK. The values of ν_G are indicated on the resistance plateaus. The resistance for $\nu = 8$ has been subtracted.

All this can be transposed to the fractional quantum Hall effect regime.

2.2 Edge states in the fractional quantum Hall regime

We will assume a spin polarized system and the first orbital Landau level partially filled : $n_s < n_\Phi$ or $\nu < 1$. Before describing fractional edge states, we will briefly present some general characteristics of the Fractional Quantum Hall effect.

Because of Landau level degeneracy, at partial filling there is a large freedom to occupy the quantum states, i.e. to fill the plane with electrons. However electrons interact and the Coulomb repulsion will reduce our freedom to distribute electrons in the plane. Let us first consider the limit of infinite magnetic field when the filling factor goes to zero. The Gaussian wavefunctions describing the cyclotron motion shrink to zero. Electrons being like point charges behave classically (no overlap between quantum states) and minimize their energy to form a crystalline state (analogous to the electron crystal observed in dilute classical 2D electron systems in zero field). The Landau level degeneracy is broken and a unique ground state is formed. In the present case, weaker magnetic field, i.e. ν not too small, the wavefunctions overlap. Electrons can not be localized to a lattice but instead will form a correlated quantum liquid. For some magic filling factors, interactions will break efficiently the Landau level degeneracy to form a *unique* collective wavefunction minimizing the energy. The magic filling factors are found to be odd denominator fractions: $\nu = 1/3$,1/5, 2/3, 2/5, 3/5, 2/7, ... [15].

2.2.1 The Laughlin states

The ground state separated from a continuum of excitations by a gap Δ is described by a unique collective wavefunction. For $\nu = 1/(2s+1)$, s integer, Laughlin proposed a trial wavefunction for the ground state which was found very accurate. The wavefunction is built from single particle states in the cylindrical vector potential gauge. Using a representation of electron coordinates as $z = x + iy$ in unit of magnetic length l_c, the single particle states in the first Landau level are :

$$\varphi_m = \frac{1}{\sqrt{2\pi 2^m m!}} z^m \exp(-|z|^2) \tag{9}$$

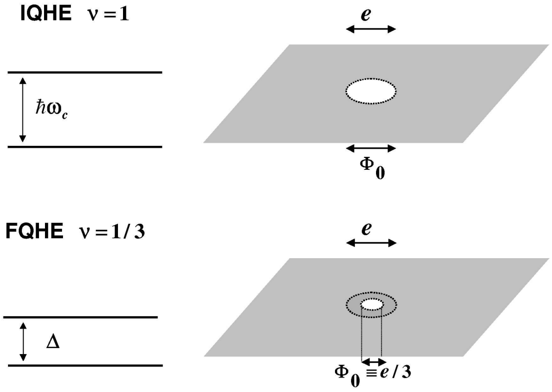

Figure 4: the introduction of an extra flux quantum in a QHE fluid leaves a hole in the collective wavefunction. In the IQHE the associated charge is e while in FQHE is it $e/3$ for $\nu = 1/3$

It is instructive to look first at the Slater determinant of electrons at filling factor 1 which is a Vandermonde determinant. Its factorization gives the following wavefunction, up to a normalization constant:

$$\Psi_1 = \prod_{i<j\leq N} (z_i - z_j) \exp(-\sum_{i=1,N} |z_i|^2) \tag{10}$$

The polynomial part ensures a uniform distribution of electrons in the plane with one state, or equivalently one flux quantum, per electron on average. The zeros at $z_i = z_j$ reflects the Pauli principle and their multiplicity 1, the Fermi statistics. At filling factor $1/(2s+1)$ the polynomial for each z_i should be of degree of $(2s+1)(N-1)$ such that all electrons are also uniformly distributed on the $(2s+1)N$ states available. A uniform distribution of electrons in the plane requires a very symmetrical polynomial. Also Laughlin proposed the simple polynomial form [3]:

$$\Psi_{1/2s+1} = \prod_{i<j\leq N} (z_i - z_j)^{2s+1} \exp(-\sum_{i=1,N} |z_i|^2) \tag{11}$$

The correlation energy is efficiently minimized by the multiplicity $2s+1$ of the zeros which ensures that electrons keep away from each other. By exchanging two electrons the wavefunction is multiplied by $(-1)^{(2s+1)} = -1$. The requirement that electrons must obey Fermi statistics is satisfied. But there is more: the extra factor $(-1)^{2s}$ expresses the fact that moving two electrons around each other adds an extra phase. This phase can be viewed as the Aharanov-Bohm flux of *two* fictive flux quanta bound to each electron. This is at the origin of the composite Fermion picture mentioned below which allows to generate more complex fractions. One can also say that electrons obey a *super* exclusion principle where each particle occupies $2s+1$ quantum sates (i.e. the area of $2s+1$ flux quanta) so minimizing the interaction (there are deep connections with the concepts of exclusonic statistics and anyonic statistics [5]).

One can show that the excitations above the ground state present a gap. The meaning of the excitations is particularly clear in the case of the best known state occurring at $\nu = 1/3$. The ground state corresponds to uniform distribution of electrons, one electron per area occupied by three flux quanta. The unique wavefunction cannot be continuously deformed and the only way to decrease the density is to empty a single particle quantum state, i.e. to create a hole having the area occupied by a single flux quanta, see Fig. 4. This can be realized by multiplying the Laughlin wavefunction by $\prod_{i=1,N}(z_i - z_h)$ where z_h is the position of the hole. The so called quasi-hole carry a charge $e^* = -e/3$. The energy cost Δ_h can be obtained by estimating the energy required to create a disc of size Φ_0/B and charge $e/3$: $(4\sqrt{2}/3\pi)(e/3)^2/4\pi\varepsilon\varepsilon_0 l_c$. Similarly quasi-electron excitations with charge $e/3$ are possible and correspond to removing a flux quantum to locally increase the electronic density with an energy Δ_e. Quasi-electron or hole excitations with charge $\pm e/q$ can be generalized for other filling factor $\nu = p/q$ with q odd. The excitation gap for quasi-electron quasi-hole pairs has been numerically estimated and calculations agree with a value $\Delta \simeq 0.092e^2/4\pi\varepsilon\varepsilon_0 l_c$ for $q = 3$. It does not depend on p (as far as spin polarized electrons are considered).

An interesting theoretical issue is the statistics associated with the excitations. It can be shown that when moving adiabatically two quasi-holes around each other and exchanging their positions, the collective wavefunction picks up a Berry's phase factor $\exp(i\pi/(2s+1))$. The excitations are not bosons nor fermions but obey a so-called anyonic statistics, a concept first introduced by Wilzeck in the context of particle physics.

2.2.2 Composite Fermions

Following the work of Jain [6], a hierarchy of the fractional filling factors can be made using the concept of Composite Fermions as a guide. This hierarchy followed more pioneering work made by Halperin [17] using a different approach to built higher order fractions from the basic $1/(2s + 1)$ states. The concept is based on statistical transmutation of electrons in 2D (or 1D). Topological considerations show that purely 2D particles are not necessarily bosons or fermions but may have any intermediate statistics (an example is the Laughlin quasi-particles). For the same reason, it is easy to "manipulate" the statistics of 3D particles such as electrons which are Fermions provided they are forced to live in 2D (or 1D). This can be done by attaching an integer number of fictive flux quanta to each electron. The price to pay is a redefinition of the wavefunction and of the Hamiltonian. An even number of flux quanta will transform Fermions into Fermions while an odd number will transmute Fermions into Bosons. In the first case we have Composite Fermions (CF) while in the second case Composite Bosons (CB). Both approaches have been used in the FQHE context. Both have their own merit and a bridge between them is possible. CF are believed to be appropriate for high order fractions and to describe the remarkable non Quantum Hall electronic state found at $\nu = 1/2$. CB make an interesting correspondence between the $\nu = 1/2s + 1$ states and superfluidity [7].

By attaching $2s$ flux quanta to each electron with a sign opposite to the external magnetic field flux, the resulting CF experience a reduced mean field. A mapping can then be done between FQHE states and IQHE states. As an example, for $s = 1$, the mean field attached to the "new electrons", the composite Fermions, is equivalent and opposite to the magnetic field $B_{1/2}$ at $\nu = 1/2$ [18]. The field experienced by the CF is thus $B_{CF} = B - B_{1/2}$. A filling factor $\nu = 1/3$ for electrons corresponds to a CF filling factor $\nu_{CF} = 1$. Similarly $\nu = p/(2p + 1)$ becomes $\nu_{CF} = p$. This describes a series of fractions observed between 1/2 and 1/3. For fields lower than $B_{1/2}$, $\nu = p/(2p - 1)$ also becomes $\nu_{CF} = (-)p$ and this describes fractions from 1 to 1/2. In general attaching $2s$ flux quanta to electrons describe the fractions $\nu = p/(p.2s \pm 1)$. The following table shows the correspondence for $2s = 2$:

ν	1/3	2/5	3/7	...	1/2	...	3/5	2/3	1
ν_{CF}	1	2	3	...	∞	...	3	2	1

The composite fermion picture is supported by experimental observations. The symmetric variations of the Shubnikov-de Has oscillations around $B_{1/2}$ are very similar to that observed around $B = 0$. We should emphasize that this is not a real cancellation of the external field, as the Meissner effect in superconductivity is, but the phenomenon is a pure orbital effect due to the $2s$ flux attachment. Convincing experiments have shown that the quasiparticles at $\nu \simeq 1/2$ behave very similarly to the quasiparticles at zero field ([19]; see also [20]).

The composite fermion picture can be used as a guide to understand multiple fractional edge channels.

2.2.3 Fractional edge channels

The picture of edge channels can be extended to the fractional case. Now the gap $\hbar\omega_c$ has to be replaced by the gap $\Delta_e + \Delta_h$ of the FQHE. Lets consider for example a filling factor $\nu = p/(2p+1)$ in the bulk, i.e. p composite fermion Landau levels filled. Using this correspondence, the formation of fractional edge channels is equivalent to that described previously for the integer Quantum Hall

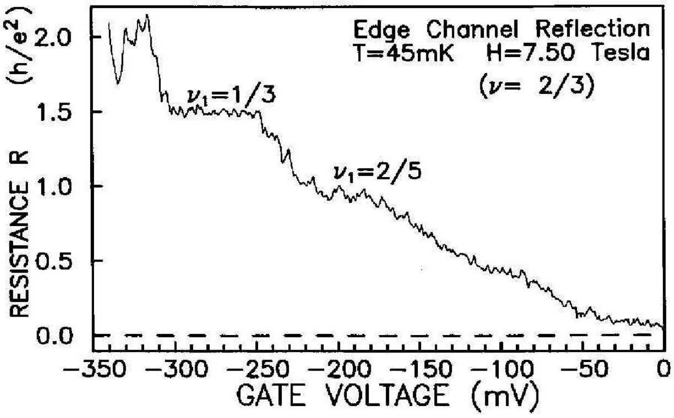

Figure 5: Fractional edge channel reflection observed for $\nu = 2/3$. The longitudinal resistance quantization indicates $\nu_G = 2/5$ and $1/3$ fractional channels. The resistance for $\nu = 2/3$ has been subtracted.

effect. Moving from the bulk to the edge, each time a CF landau level crosses the Fermi energy a line of gapless excitation is built. These defines p chiral fractional edge channels (the last one corresponding to the 1/3 edge channel).

The CF approach for edge channels is convenient for pedagogical presentation and gives certainly a fair qualitative representation, but is certainly not complete. Including screening of the external potential in a Thomas Fermi approach is a first step to improve quantitatively the description [21] but this does not change qualitatively the overall picture. A important physics not included in this approach is the Luttinger liquid properties described below: it changes the transport properties. The hierarchy of fractional edge channels which can be derived in the Luttinger liquid approach coincide with that of the CF approach.

Experimentally, the existence of fractional edge channels can be probed in transport experiments using the reflection induced by a QPCs in a manner similar to the integer case. This is shown in Fig.5. Here the filling factor in the leads is $\nu = 2/3$ and the access resistance $3h/2e^2$ has been subtracted. The plateaus associated with the refection of the 2/5 and 1/3 edge channels are clearly observable.

The picture described here is expected to apply to smooth edges, as it is the case in ordinary samples. Another approach has been proposed for hard wall confinement in Ref.[22].

2.3 Fractional Edge Channels as Luttinger liquids

The tunnel transfer of an electron from a metallic contact to a $\nu = 1/3$ FQH liquid involves the transformation of an electron into three quasiparticles. The number of possibilities to choose 3 quasiparticles in the range eV and satisfying the energy conservation required for elastic tunneling increases rapidly with V, this immediately implies that the tunneling I-V characteristics should be non-linear. So, the fractional edge states should not behave has an ordinary Fermi liquid for which linear conduction is expected. On a different approach, a similar conclusion is obtained in the Luttinger liquid description which uses bosonic collective charge mode on the fractional edge. This is what we will describe below.

2.3.1 Hydrodynamical approach of fractional edge states

X.G. Wen [23] has first shown the deep connection between fractional edge channels and the concept of Tomonaga-Luttinger liquids[24, 25] . We will here repeat the phenomenological hydrodynamical approach of Wen in the simple case of a Laughlin state in the bulk, filling factor $\nu = 1/2s + 1$. We will start with a classical approach and keep only incompressibility as a quantum ingredient.

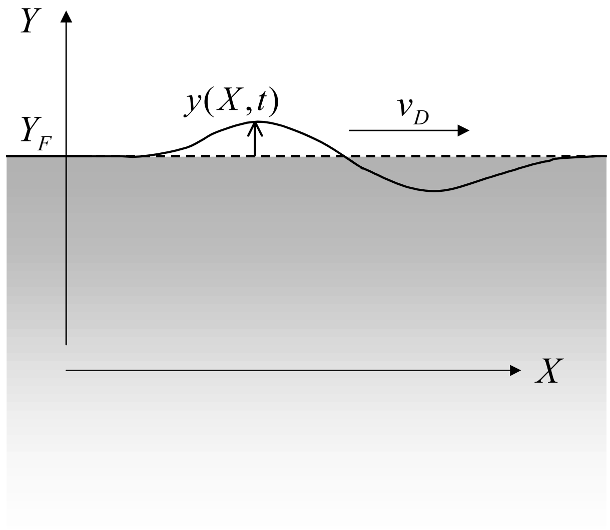

Figure 6: Wen's approach for the Chiral Luttinger liquid picture of a fractional edge channel at $\nu = 1(2s+1)$. The incompressibility of the FQHE fluid allows only periphery deformations $y(X,t)$ propagating at the drift velocity.

The only possible excitations are periphery deformations of the 2D quantum Hall conductor which preserves the total area (like a 2D droplet of an ordinary liquid). This is shown schematically in Fig.6 . If we denote $y(X,t)$ the deformation of the boundary located at position $Y = Y_F$, the time varying electron density is given by :

$$n(X, Y, t) = n_s \Theta(Y - Y_F - y(X, t)) \tag{12}$$

where $n_s = \nu eB/h$ and $\Theta(x)$ is the Heaviside function. We wish to find the equations of motion for y. To do that we have to remind that, within the first Landau level, the single particle motion is given by the reduced Hamiltonian $H_1 = \frac{1}{2}\hbar\omega_c + U(Y)$ and the coordinates X and Y are conjugate with Poisson's bracket $\{X, Y\} = 1/eB$. Using the equation of motion for the 2D density: $\partial n/\partial t + \{H_1, n\} = 0$ we get the equation describing the chiral propagation of the shape deformations y at $Y = Y_F$:

$$\partial y/\partial t + v_D \partial y/\partial X = 0 \tag{13}$$

where $v_D = \frac{1}{eB} \left| \partial U/\partial Y \rfloor_{Y=Y_F} \right|$ is the drift velocity.

The potential energy associated with the deformation is

$$U = \frac{1}{2}\int dX n_s y^2 \frac{\partial U}{\partial Y} = \frac{\hbar v_D}{\nu\pi}\int dX \left(\pi n_s y\right)^2 \tag{14}$$

If we define $\widetilde{\phi}$ the charge variation integrated on the upper edge in units of π as follows:

$$\widetilde{\phi} = \pi \int_{-\infty}^{X} n_s y dX$$

and the excess charge density (per unit length) $\widetilde{\rho}$ by:

$$\widetilde{\rho} = \frac{1}{\pi}\frac{\partial\widetilde{\phi}}{\partial X} \tag{15}$$

we get the action:

$$S = -\frac{\hbar}{\pi\nu}\int dX dt \frac{\partial\widetilde{\phi}}{\partial X}\left(\frac{\partial\widetilde{\phi}}{\partial t} + v_D \frac{\partial\widetilde{\phi}}{\partial X}\right) \tag{16}$$

and the Hamiltonian is $\mathcal{H} = U$:

$$\begin{aligned} \mathcal{H} &= \partial\widetilde{\phi}\partial X\Big)^2 && (17) \\ &= \frac{hv_D}{2\nu}\int dX\,(\widetilde{\rho})^2 && (18) \end{aligned}$$

So far the model is purely classical. By defining the conjugate of $\widetilde{\phi}$ as $\widetilde{\pi} = -\frac{\hbar}{\pi\nu}\partial\widetilde{\phi}/\partial X$ and we can quantize the fields using:

$$\left[\widetilde{\pi}(X), \widetilde{\phi}(X')\right] = i\hbar\delta\,(X - X') \quad (19)$$

At first sight, the dynamic of the bosonic modes describing the periphery deformations seems not contain more physics than that of phonons or photons. The non trivial physics arises when adding from outside an electron to the edge or removing a Laughlin quasiparticle to transfer it to the opposite edge, as it is the case in tunneling experiments. Such operation involves an infinite number of bosonic modes. This is at the origin of strong non-linearities in the transport properties, a property not shared by ordinary Fermi liquids.

By definition, the creation operator $\psi^\dagger$ for one electron on the upper edge satisfies:

$$\left[\widetilde{\rho}(X), \psi^\dagger(X')\right] = \delta(X - X')\psi^\dagger(X') \quad (20)$$

On the other hand, the 1D excess density $\widetilde{\rho}$ is related to the conjugate of $\widetilde{\phi}$ by $\widetilde{\pi} = -\frac{\hbar}{v}\widetilde{\rho}$ and we have:

$$\left[\widetilde{\rho}(X), \widetilde{\phi}(X')\right] = -i\nu\hbar\delta\,(X - X') \quad (21)$$

which immediately implies :

$$\psi^\dagger \propto \exp(i\widetilde{\phi}/\nu) \quad (22)$$

$\psi^\dagger$ creates a unit charge at X but it is not an electron operator unless it satisfies Fermi statistics. Exchanging two electrons at position X and X' gives $\psi^\dagger(X')\psi^\dagger(X) = \exp(-i\frac{\pi}{\nu}sgn(X - X'))\psi^\dagger(X)\psi^\dagger(X')$. The requirement that the bare particles are Fermions implies

$$\nu = 1/(2s + 1) \quad (23)$$

The beauty of Wen's hydrodynamical approach is that the series Laughlin filling factors appear $1/(2s + 1)$ naturally as a consequence of incompressibility and Fermi statistics.

To obtain more fractional filling factors, one must introduce additional bosonic modes at the periphery (for example: p modes for $p/(2ps + 1)$ which is consistent with the composite fermion approach.

Finally, one can define similarly the quasiparticle operator which creates a charge $1/(2s + 1)$ on the edge:

$$\left[\widetilde{\rho}(X), \psi^\dagger_{qp}(X')\right] = \nu\delta(X - X')\psi^\dagger_{qp}(X') \quad (24)$$

which writes as

$$\psi^\dagger_{qp} \propto \exp(i\widetilde{\phi}) \quad (25)$$

It shows *fractional* statistics $\psi^\dagger_{qp}(X')\psi^\dagger_{qp}(X) = e^{(-i\pi\nu sgn(X-X'))}\psi^\dagger_{qp}(X)\psi^\dagger_{qp}(X')$ as do a Laughlin quasiparticle.

The above set of equations for the electron operator $\psi^\dagger$ and for the bosonic modes are characteristics of those of a Luttinger liquid. Because of the direction of propagation imposed by the magnetic field (no counter propagating mode on the same edge) it is called a Chiral Luttinger Liquid. The conductance $\nu e^2/h$ correspond to the conductance ge^2/h of a Luttinger Liquid and one usually identifies $g = \nu$. As for Luttinger liquids there is an algebraic decay of the correlation functions.

We have $\left\langle \widetilde{\phi}(X,t)\widetilde{\phi}(0,0) \right\rangle = \langle 0| e^{i\mathcal{H}t}\widetilde{\phi}(X)e^{-i\mathcal{H}t}\widetilde{\phi}(0)|0\rangle = const. - \nu \ln(X - v_D t)$ and the time ordered single-particle Green's function: $\langle 0| T\left\{\psi^{\dagger}(X,t)\psi(0,0)\right\}|0\rangle = \exp\left(\frac{1}{\nu^2}\left\langle \widetilde{\phi}(X,t)\widetilde{\phi}(0,0)\right\rangle\right)$ decreases as $(X - v_D t)^{-1/\nu}$. For 1/3 one see that this is the product of 3 Green's functions, reminiscent from the fact that an electron has to fill 3 states (or excite 3 quasiparticles). This gives a Tunneling Density of State (TDOS) for electrons injected at energy ε above the Fermi energy E_F which decreases as $\sim |\varepsilon - E_F|^{(1/\nu-1)}$. This implies that, for electrons tunneling between a Fermi liquid and a chiral Luttinger fractional edge channel, the finite temperature tunnel conductance $G(T)$ and the zero temperature differential conductance dI/dV show the following power laws:

$$G(T) \sim (T/T_B)^{\gamma} \tag{26}$$

$$\frac{dI}{dV} \sim (V/V_B)^{\gamma} \tag{27}$$

$$\gamma = \frac{1}{\nu} - 1 \tag{28}$$

where T_B and V_B are related to the coupling energy of the tunnel barrier. Power laws characterizing the chiral Luttinger liquid in the Fractional Quantum Hall regime have been experimentally observed (see below).

The chirality leads to some differences with ordinary Luttinger Liquids for which $1/\nu$ is to be replaced by $(g^{-1}+g)/2$. In 1D, g is related to the strength of a short range interaction which can take arbitrary values and the relation: $\gamma = (g + g^{-1} - 2)$ always holds. A *continuous variation* of the $g = \nu$ parameter is a priori not expected in the FQHE regime because the magnetic field stabilizes special fractional values of ν in the bulk (as the Jain's series). A generalization of Wen's approach for $\nu = p/(2sp+1)$ shows that one must have p branches of bosonic modes (consistent with the CF picture). These branches interact together and give a relation between the tunneling exponent γ and ν not simply given by 28. For the simplest series of Jain's filling factor $p/(2p \pm 1)$ between 1 and 1/3 , the exponent γ is expected to be

$$\gamma = \frac{2p+1}{p} - \frac{1}{|p|} \tag{29}$$

i.e. constant ($\gamma = 2$) between filling factor 1/2 and 1/3 and decreasing linearly ($\gamma = 1/\nu \sim B$) form 2 to 1 between filling factor 1/2 and 1.

2.3.2 Experimental evidence of chiral Luttinger liquids

Tunneling electrons from a metal to the edges: The best evidence for Luttinger liquid properties is obtained by probing the tunneling density of states (TDOS). To do that, measurements have to be *non-invasive*, i.e. a weak tunnel coupling is required. Indeed, a tunneling experiment measures the TDOS only if higher order tunneling process are negligible, which means small transmission and small energy. At large energy, the current varies $\sim |\varepsilon - E_F|^{\gamma+1}$ and so the effective coupling will increase with voltage $\varepsilon \equiv eV$ or temperature $\varepsilon \equiv k_B T$.

Convincing experiments have been performed by the group of A.M. Chang [26, 27, 28]. The tunnel contact is realized using the cleaved edge overgrowth technique. By epitaxial growth on the lateral side of a 2DEG, a large tunnel barrier is first defined followed by a metallic contact realized using heavy doped semiconductor. The advantage is weak coupling and high enough barrier (to disregard change of the transparency when applying a large voltage). Also, probably important is the fact that the metallic contact close to the edge provides screening of the long range Coulomb interaction (short range is needed for having power laws).

Fig.7 shows example of I-V characteristics: a power law of the current with applied voltage $I \sim V^{\alpha}$, $\alpha = \gamma + 1$, is well defined over several current decades for $\nu = 1/3$. The exponent α found is 2.7-2.65 close to the value 3 predicted by the theory for $\gamma = \frac{1}{\nu} - 1$. For other filling factors similar algebraic variations are also observed. In the same figure, the tunneling exponent deduced from a series of I-V curves is shown as a function of the magnetic field or $1/\nu$. For filling

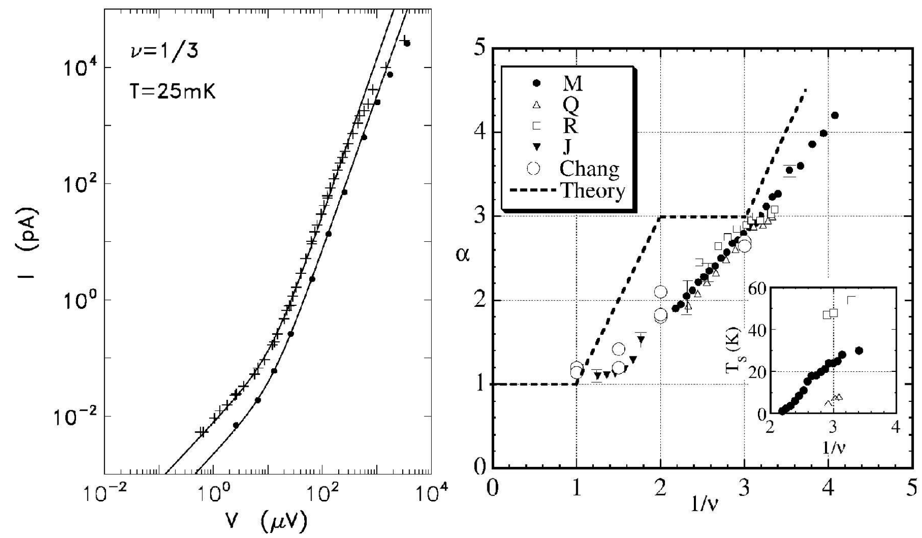

Figure 7: Left : Log-Log plot of an I-V curve at $\nu = 1/3$ clearly shows the algebraic variation of current with voltage which characterizes a Luttinger liquid. Right : the exponent α is plotted versus $1/\nu$ and compared with theoretical predictions (adapted from Ref.) .

factor $1/2 < \nu < 1/3$ the constant exponent predicted [29, 30, 31] is not observed and instead the exponent varies rather linearly with field or $1/\nu$. However, some experiments made with the cleanest samples have shown signs of a plateau in the exponent in a narrow filling factor value near 1/3.

Theoretical attempts to explain quantitatively the discrepancies have been made. Taking into account the long range Coulomb interaction slightly lowers the exponent. The modified Luttinger liquid theories can also include the finite conductivity in the bulk for non fractional filling factors. Indeed a finite conductivity modifies the dispersion relation of the bosonic chiral modes and so the exponents. With reasonable parameters these modifications are not yet able to fully reproduce the data [30, 31]. The discrepancy between experiments and predictions may be due to the reconstruction of the edge. Wen's model assume a sharp density variation at the edge of the 2D sample. In real samples, the density decreases smoothly and some additional edge states corresponding to filling factor lower than the bulk filling factor may also strongly modify the exponents. A recent work by Mandal and Jain shows that taking into account interactions between composite fermions chiral edges may lead to a continuous variation of the exponent[32]. The reader will find more in recent review made by A.M. Chang [33]

Tunneling between edges : Fractional Edge channel with an artificial impurity: Transport experiments between two fractional edges can be done using an artificial impurity, a Quantum Point Contact. Contrary to previous experiments where electrons were injected from an ordinary metal (Fermi liquid) and the coupling was weak, we can probe here the transfer of charge *through* the FQHE fluid. In particular there is no restriction on the nature of the charge (obviously e in the previous case) while they can be fractional here. For quantitative comparison to theory, this strategy however is less reliable than the previous one. For finite voltage difference V_{ds} applied across the QPC to induce a current, the shape of the scattering potential can change. This can induce a trivial variation of the transmission with V_{ds} which can make identification of power laws difficult. Measurements are thus reliable only at very low temperature (<100mK) and small voltages (<100μV) for comparison with theories.

strong barrier: We first consider the case where barrier is high and a tunnel barrier is formed (so-called pinch-off regime). Electrons are strongly backscattered at all energies and the tunneling current is weak. The results are expected similar to that obtained in A.M. Chang's experiments.

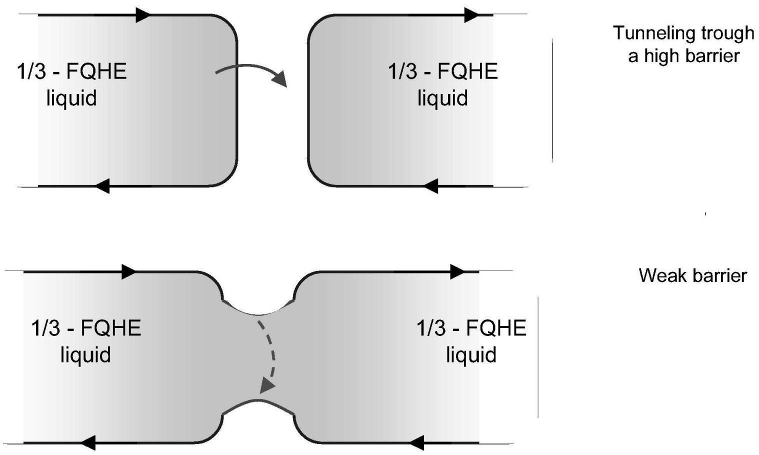

Figure 8: Schematic view of charge transfer in the case of a strong barrier (upper figure) and a weak barrier. In the later case the FQHE fluid is weakly perturbed and charge transfer occurs via the FQHE fluid.

Tunneling occurring between two fractional edge (and not between a metal and a fractional edge) the dI/dV_{ds} characteristics will be proportional to the *square* of the TDOS. The exponents for the conductance is doubled. For example $\gamma = 4$, i.e. $2.(\frac{1}{\nu} - 1)$ for $\nu = 1/3$. This approach has been used by several groups. A difficulty is that sample inhomogeneities around the QPC may lead to transmission resonances difficult to control. Ref.[34] exploits the Luttinger predictions for tunneling through such a resonant state. A further difficulty is the high value of the power law for the conductance with temperature or voltage which is is expected to be measurable only at very low conductance. For $\nu = 1/3$, for example, exact finite temperature calculations of IV characteristics, see below, shows that the exponent 4, becomes the dominant term only when the conductance is smaller than $10^{-4}e^2/3h$ [35] . Otherwise an effective exponent, much smaller than 2 is observed. Up to now, no experimental group have tried to do measurements in this limit.

weak barrier: The regime where the barrier is very weak is more interesting. Practically, the QPC gently pushes the upper edge close to the lower edge to induce a quantum transfer of particles from one edge to the other. The QPC potential is weak enough to not make appreciable change of the local filling factor. A characteristic signature of the Luttinger liquid physics is the *vanishing* transmission at low temperature or bias voltage even in the case of a so weak coupling that the transmission would be close to 1 in absence of interaction. The low energy strong backscattering limit continuously evolves toward a weak backscattering limit at large energy (large transmission). In this regime, we will see later that integer charges tunnel at low energy, while fractional charges tunnel at large energy.

To describe the tunneling between the upper and lower edge of Fig.8 we introduce bosonic modes $\widetilde{\rho}$ and $\widetilde{\phi}$ previously derived with the subscript $+/-$ for the upper and lower modes respectively. Without coupling by the artificial impurity, they are independent and the Hamiltonian is the sum of their Hamiltonian. The impurity of strength λ situated in $X = 0$ induces a coupling between the excitations: $\simeq \psi^{\dagger}_{qp,+}\psi_{qp,-} + \psi^{\dagger}_{qp-}\psi_{qp+}$ which gives the interaction term:

$$\mathcal{H}_{int} = \lambda \cos(\phi_{+}(0) - \phi_{-}(0)) \tag{30}$$

At low energy, the system flows to an insulating state and the conductance displays the same power law with T or V_{ds} than the one expected for a strong impurity potential (tunnel barrier)

$$G \sim \frac{e^2}{3h}\left(\frac{\varepsilon}{T_B}\right)^{2(\frac{1}{\nu}-1)} \rightarrow 0 \;\; \text{for } \varepsilon \ll T_B \tag{31}$$

where T_B is an energy scale related to the impurity strength λ. At large energy, a conductance close but smaller than the quantum of conductance $\frac{e^2}{3h}$ is recovered. The Luttinger liquid theory predicts

$$G = \frac{e^2}{3h} - G_B \quad \text{with } G_B = \frac{e^2}{3h}\left(\frac{\varepsilon}{T_B}\right)^{2(\nu-1)} \to 0 \quad \text{for } \varepsilon \gg T_B \tag{32}$$

G_B is called the backscattering conductance. If we call I the forward current, $I_0 = \frac{e^2}{3h}V_{ds}$ the current without impurity, the current associated with particles backscattered by the impurity is $I_B = I_0 - I$ from which one can define $G_B = I_B/V$. The above formula correspond to *strong* and *weak* backscattering limits. In the first case there is a weak tunneling of particles between the left and right side, while in the second case there is a weak quantum transfer of particles between the upper edge and the lower edge. There is an interesting *duality* with $\nu \longleftrightarrow 1/\nu$.

Conformal field theories have been used to exactly solve the problem of a Luttinger liquid with one impurity providing a continuous description between both limits [36, 37]. The so-called FLS theory exploits the charge conservation when the particle are scattered by the impurity centered in $X = 0$. By defining the even and odd charge modes (and corresponding fields):

$$\widetilde{\rho^e}(X,t) = \frac{1}{\sqrt{2}}\left(\widetilde{\rho_+}(X,t) + \widetilde{\rho_-}(-X,t)\right) \tag{33}$$

$$\widetilde{\rho^o}(X,t) = \frac{-1}{\sqrt{2}}\left(\widetilde{\rho_+}(X,t) - \widetilde{\rho_-}(-X,t)\right) \tag{34}$$

for which the variable X is now limited to the semi-infinite line $X \leq 0$.

The Hamiltonian to consider reduces to:

$$\mathcal{H} = \frac{hv_D}{2\nu}\int_{-\infty}^{0} dX \left[\widetilde{\rho^o}(X,t)^2 + \lambda_1 \cos\left(\sqrt{2}\widetilde{\varphi^o}(0)\right)\right] \tag{35}$$

while the even mode is decoupled.

The equation is very similar to a Sine-Gordon equation (SG) but with the SG term only at the boundary, while $\widetilde{\rho^o}$ is solution of a free propagation equation (velocity v_D) for $X < 0$. Classically, it is easy to show that this boundary SG equation admits solutions using a combination of the natural kink and anti-kink of the ordinary SG equation. By definition, a kink (or a soliton) in the field (of the charge density) which is solution of the ordinary SG equation propagates without deformation and is also solution of the free propagation equation for $X < 0$. By linearity, superpositions of kink and anti-kinks are also solutions. The effect of the boundary term is mainly to convert kink into antikink. Physically, the effect of scattering is that a positive pulse of charge can be reflected as a negative pulse. The step from classical to quantum integrability is made using conformal field theories [36, 37]. One can show that applying a voltage bias V_{ds} between reservoirs emitting electrons in the upper and lower edges is, in the convenient basis for interacting electrons, equivalent to send a regular flow of kink which are randomly transformed into antikink. Kink and antikink respectively contribute to the forward and backscattered current. The Landauer formula adapted to this approach gives the backscattering current which expresses simply as :

$$I_B(V_{ds}, T_B) = ev_D \int_{-\infty}^{A(V_{ds})} d\alpha \rho_+(\alpha)\left|S_{+-}(\alpha - \alpha_B)\right|^2 \quad \text{and} \quad I = \nu\frac{e^2}{h}V_{ds} - I_B \tag{36}$$

were $\rho_+(\alpha)$ (not to be confused with previous notations) is the density of incoming kink at energy parametrized by e^α, and

$$\left|S_{+-}(\alpha - \alpha_B)\right|^2 = \frac{1}{1 + \exp\left[2(1-\nu)(\alpha - \alpha_B)/\nu\right]} \tag{37}$$

is the probability for kink to anti-kink conversion (the scattering probability) with α_B related to the impurity strength T_B. A series expansion in T_B/V_{ds} and V_{ds}/T_B for respectively weak and

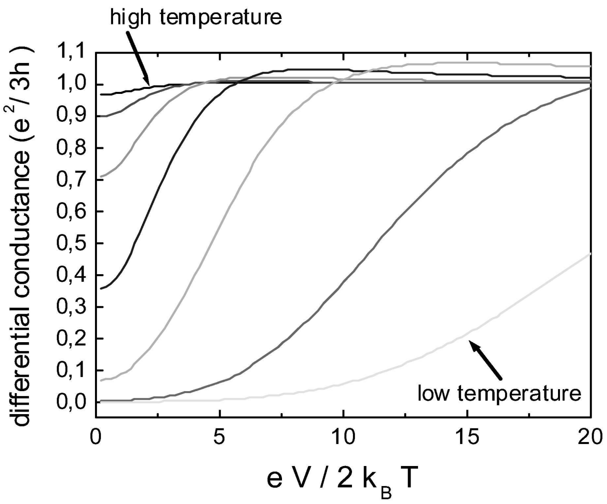

Figure 9: Theoretical curves for the differential conductance versus voltage calculated for different values of the ratio T/T_B. The numerical exact solution of Ref.[36, 37] is used. The conductance is a universal function of the variable T/T_B and $eV_{ds}/2\pi k_B T$.

strong backscattering gives the current where all coefficients are known analytically

$$I_B = \nu \frac{e^2}{h} V_{ds} \sum_n \nu a_n(\nu) \left(\frac{V_{ds}}{T_B}\right)^{2n(\nu-1)} \quad \text{for} T_B/V_{ds} < 1 \tag{38}$$

$$I = \nu \frac{e^2}{h} V_{ds} \sum_n a_n(\frac{1}{\nu}) \left(\frac{V_{ds}}{T_B}\right)^{2n(\frac{1}{\nu}-1)} \quad \text{for} V_{ds}/T_B < 1 \tag{39}$$

The curve $I(V_{ds})$ describing the whole transition from strong to weak backscattering can be calculated. Note again the duality $\nu \longleftrightarrow 1/\nu$. For finite temperature numerical solutions are also available giving the whole information $I(V_{ds}, T)$.

For finite temperature, one can show that the conductance is a function of the reduced variable T/T_B and $eV_{ds}/2\pi k_B T$:

$$G(T, V_{ds}) = \frac{e^2}{3h} f\left(\frac{T}{T_B}, \frac{eV_{ds}}{2\pi k_B T}\right) \tag{40}$$

and measuring $G(T, 0)$ fixes the only parameter T_B. The V_{ds}/T scaling law also can be tested very accurately. Fig.9 shows theoretical calculations of the differential conductance for various values of the parameter T_B (the Bethe ansatz method for calculating the kink-antikink distribution at finite temperature has been used for the numerical calculation following the results of Ref.[36, 37]. We can see that, increasing the energy (the voltage or the temperature), leads to a progressive transition from the strong backscattering regime to the weak backscattering regime.

Fig.10 shows data obtained in our group for the conductance in the strong backscattering regime for $\nu = 1/3$. Experiments are made in the intermediate regime (i.e. $G > 10^{-4}e^2/3h$). The effective exponent deduced from a series a dI/dV_{ds} curve for different impurity strength is compared with the effective one calculated using the finite temperature exact solution. The agreement is rather good. The theoretical graph in inset of the figure shows that the asymptotic scaling exponent $2(\nu^{-1} - 1) = 4$ is not expected except for conductance lower than 10^{-4}, which is experimentally difficult to obtain. It is important to say that there are no adjustable parameters.

Indeed, there are still many open problems for a quantitative description of conductance measurements using the Luttinger liquid model. Long range interactions are one of this. One can show that the dispersion relation bosonic chiral edges modes which usually varies linearly with the wavenumber k get a contribution $k \ln(k)$. Such contribution is known from edge magneto-plasmon radiofrequency experiments realized in classical or in quantum Hall 2D electron systems where the

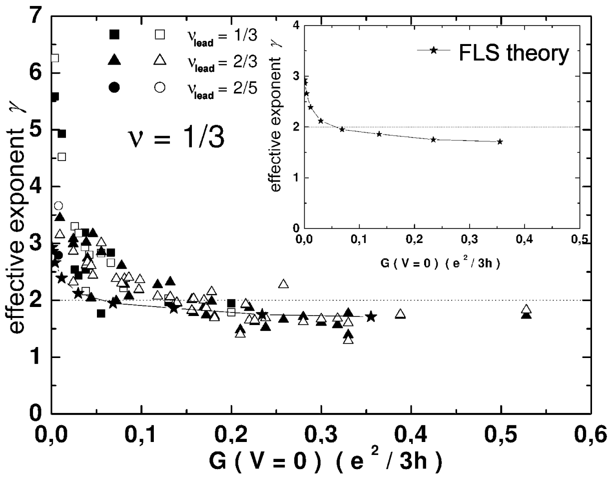

Figure 10: Exponent of the algebraic variation of the differential conductance with voltage measured in the strong backscattering regime versus the zero bias conductance normalized to $e^2/3h$. The solid line is a comparison with the FLS predictions. The scaling exponent $\alpha = 2(\nu^{-1} - 1) = 4$ is only expected in a regime of extremely low conductance (10^{-4}).

neutral collective modes are excited and resonantly detected. When the energy is low enough such that the wavelength is larger than the width of the sample, the Coulomb interaction couple the edges. The power law of the TDOS is lost. Instead the TDOS is expected to vary with energy like $\exp(const \times (\ln \varepsilon)^{3/2})$ [38, 39, 40].

3 Fractionally charged carriers

Can an electrical current be carried by fractional charge? From quantum electrodynamics and charge conservation, it is known that the total charge of an isolated body should always be an integer number in units of e. While particles propagating freely in the vacuum are restricted to integer charge, no such absolute requirement is imposed in condensed matter to quasiparticles, the elementary excitations above the ground state which carry the current. They are the product of a complex collective motion of many particles. The topological singularities of the wave-functions may lead to many possibilities. The solitonic excitations predicted in polyacetylene or the Laughlin charges in the Quantum Hall regime are exact fractions. Fractions manifest particular evident or hidden symmetries. The quasiparticle charge may not be restricted to exact mathematical fraction: in Luttinger liquid theories, describing one-dimensional interacting systems, charge ge and $(1-g)e$ are expected where g is related to the interaction strength. It can take any value between 0 and 1.

Up to now the only experimentally realizable system able to display fractional charge carriers is the FQHE. But how to measure the charge carrying the current? Conductance is unable to probes directly the charge. It informs on the average rate of quasiparticles received by a contact which went through the conductor after they have been emitted by another contact. Conductance measures the quasiparticle transmission and, if interference are observed, it is sensitive to the wave nature of the quasiparticles. This is similar to optics where the average intensity of light tells about transmission but says nothing about the photon. To probe the graininess of the current one must make a further step and consider the fluctuations: the so-called shot noise. Measuring the fluctuation of light beams has given the direct evidence of the photon as the elementary grain of light. Similarly, measuring the electrical current fluctuations gives a direct information on the quanta of charge carrying the current.

According to Schottky [41], the random transfer of charge q across a conductor generates an average current $\overline{I}$ but also finite temporal fluctuations of the current ΔI around the mean value.

Consider an observation during a finite time τ. The current is related to the average number of transferred electrons $\overline{N}$ via $\overline{I} = q\overline{N}/\tau$, while the square of the current fluctuations are $\overline{(\Delta I)^2} = q\overline{I}/\tau \frac{\overline{(\Delta N)^2}}{\overline{N}}$. If the statistics of transfer events is Poissonian $\overline{(\Delta N)^2} = \overline{N}$ the well known Schottky formula is obtained:

$$\overline{(\Delta I)^2} = 2q\overline{I}\Delta f = S_I \Delta f \quad (41)$$

where we have introduce the effective frequency bandwidth of measurement $\Delta f = 2/\tau$ and the current noise power S_I. The noise power is directly proportional to the carrier charge q . This expresses that noise is a direct consequence of charge granularity. The simultaneous measure of the average current and its fluctuations gives a simple direct measurement of q, free of any geometrical or material parameters.

To measure shot-noise, one performs a non-equilibrium experiment and therefore probe excitations above the ground state: the quasiparticles. Also, the bias voltage across contacts has to be larger than $k_B T$ otherwise the dominant noise measured is the thermal or Johnson-Nyquist noise: $S_I = 4Gk_B T$. Johnson-Nyquist noise is an equilibrium quantity which only probes conductance and not the charge of the excitations.

3.1 Shot-noise in quantum conductors

Here we discuss the origin and properties of noise in conductors. This applies immediately to the case of the Integer QHE regime. For simplicity we consider a single mode conductor or equivalently single edge channel. An artificial impurity, for example a Quantum Point Contact, is used to induce backscattering. According to the Landauer formula the left contact injects electrons at a rate eV/h where V is the voltage applied between the left and the right contact. This leads to an incoming current $I_0 = e(eV/h)$. If $\mathcal{T}$ is the transmission through the QPC, the transmitted current is :

$$I = \mathcal{T}.I_0 = \mathcal{T}\frac{e^2}{h}V \quad (42)$$

and the backscattered current:

$$I_B = I_0 - I = (1-\mathcal{T})\frac{e^2}{h}V \quad (43)$$

One can show that the Fermi statistics is responsible of a remarkable properties: for long observation time, the incoming current appears to be noiseless (each electron arrives regularly at a frequency eV/h leading to a temporally structureless electron flow). As a result the only fluctuations arise from the binomial probability to be transmitted or reflected. The spectral density S_I of the low frequency current fluctuations is thus given by [42]:

$$S_I = 2eI_0\mathcal{T}(1-\mathcal{T}) \quad (44)$$

For multimode conductors $\mathcal{T}_n$ denoting the transmission of the n^{th} mode, the generalization is straightforward and is : $S_I = 2eI_0 \sum_n \mathcal{T}_n(1-\mathcal{T}_n)$. This prediction has been quantitatively verified in very sensitive shot noise measurement using QPC [43, 44]. A review on the remarkable low noise of quantum conductors can be found in [45]. At finite temperature, the shot noise is :

$$S_I = 2e\frac{e^2}{h}\left(\mathcal{T}^2 k_B T + \mathcal{T}(1-\mathcal{T})eV\coth\left(\frac{eV}{2k_B T}\right)\right) \quad (45)$$

At zero bias voltage the Johnson Nyquist equilibrium noise $4\left(\mathcal{T}\frac{e^2}{h}\right)k_B T$ is recovered. Above a cross-over voltage $k_B T/2e$, shot noise dominates.

Two limits are interesting to consider for the following:

- Strong backscattering regime $\mathcal{T} \ll 1$: this is the regime of Poissonian transfer from left to right of charge e.

$$S_I = 2eI \quad ; \quad I \ll I_0 \quad (46)$$

- Weak backscattering regime $1-\mathcal{T} \ll 1$: most electrons are transmitted, but there is of Poissonian transfer of "missing electrons", i.e holes, from left to right. Alternatively, in the IQHE

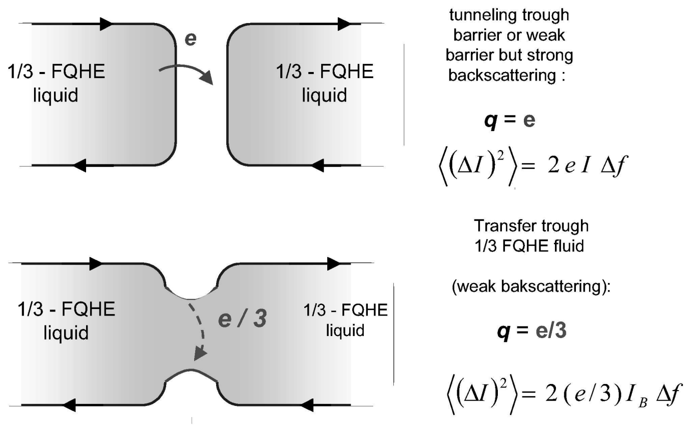

Figure 11:

regime, this can be viewed as Poissonian transfer of electron from the upper to the lower edge *via* the QHE fluid

$$S_I = 2eI_B \quad ; \quad I_B = I_0 - I \ll I_0 \tag{47}$$

3.2 Shot-noise in the fractional regime

In the limit of small transmission ($I \ll I_0$, strong backscattering) it is reasonable to expect transfer of charge e as correlations between left and right FQHE quantum fluids are reduced. However, for large transmission ($I_B = I_0 - I \ll I_0$, weak backscattering) the weak effect of the impurity will not affect the FQHE correlations, the Poissonian transfer of holes may correspond to Laughlin quasihole with fractional charges as shown schematically in Fig.11.

A complete understanding requires to include the Luttinger liquid dynamics of the fractional edge channels. This has been done in these two limiting cases in Ref.[46] and then by using the exact FLS Theory. For Laughlin filling factor $\nu = 1/(2s+1)$, including finite temperature, the strong and weak backscattering limiting cases give respectively:

$$S_I \simeq 2eI \coth(eV/2k_B\theta) \qquad I \ll I_0 = \frac{1}{3}\frac{e^2}{h}V \tag{48}$$

$$S_I \simeq 2\frac{e}{2s+1}I_B \coth(\frac{e}{2s+1}\frac{V}{2k_BT}) \qquad I_B = I_0 - I \ll I_0 \tag{49}$$

Here, the Johnson-Nyquist thermal noise contributions $4Gk_BT$ and $4G_Bk_BT$ have been subtracted respectively for clarity. In the first case only electrons are found to tunnel as expected. In the second case fractional charge excitations are found. In this limit, the noise provides a *direct* way to measure the fractional Laughlin charge $e/2s+1$.

A fractional $e/(2s+1)$ charge is also found in the argument of the coth function. However the meaning is different. The cross-over from thermal to shot noise corresponds to electro-chemical potential difference $\Delta\mu = eV/(2s+1)$ comparable to k_BT. However, this is not a measure of the fractional quasiparticle charge. This is a measure of the fractional filling of the quantum state at equilibrium, like the conductance $e^2/(2s+1)h$ is. This is only in the large voltage limit that $S_I \simeq 2(\frac{e}{2s+1})I_B$ really measures the quasiparticle charge. This is a non-equilibrium regime where quasiparticles excitations dominate over ground state properties. Nevertheless observation of a three times larger voltage for the thermal cross-over in noise experiments has been an important confirmation of Eq.49.

The zero temperature limit of expressions 48 and 49 have been also derived in [47] using Luttinger liquid in the perturbative limit. The exact solution of the FLS model presented in the previous section allows not only to calculate the current in all regimes but also to calculate the

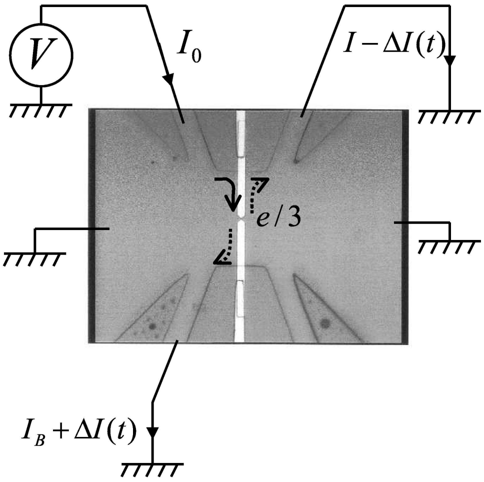

Figure 12: schematic view of the measurement. The fluctuations of the transmitted current I and of the reflected current I_B are both measured. A very fast dynamic signal analyzer calculates in real time the cross-correlation of the fluctuations. Uncorrelated noises are thus eliminated increasing the sensitivity and reliability.

noise [36]. To obtain the noise, one can mimic the wavepacket approach used by T. Martin and R. Landauer for the noise of non interacting Fermions Ref.[48] . The incoming kinks of the field $\widetilde{\phi^o}$ correspond to a regular flow of solitons in $\widetilde{\rho^o}$. The regular flow is noiseless but the random scattering of kinks into anti-kinks produces noise in the outgoing current. When $|S_{+-}(\alpha-\alpha_B)|^2 \ll 1$ this a Poissonian process while if $|S_{+-}(\alpha-\alpha_B)|^2$ is not negligible, the statistics is binomial and the fluctuations are proportional to $|S_{+-}(\alpha-\alpha_B)|^2(1-|S_{+-}(\alpha-\alpha_B)|^2)$ which plays the role of the $\mathcal{T}(1-\mathcal{T})$ factor for non-interacting electrons. The expression for the noise is thus simply

$$S_I(V) = 2e^2 v \int_{-\infty}^{A(V_{ds})} d\alpha \rho_+(\alpha) |S_{+-}(\alpha-\alpha_B)|^2 (1-|S_{+-}(\alpha-\alpha_B)|^2) \qquad (50)$$

Here $\nu = 1/(2s+1)$. Exact expressions and technical mathematical details can be found in Refs [36]. The special simple form of $|S_{+-}(\alpha-\alpha_B)|$ leads to a relation between current and noise where $S_I = \frac{v}{1-v}(V\frac{dI}{dV} - I) = \frac{v}{1-v}(I_B - V\frac{dI_B}{dV})$. From it, using the weak and strong backscattering limits of the Luttinger theory, we can easily check that $S_I \to 2(\nu e I_B)$ and $2eI$ respectively in agreement with the zero temperature limit of 48 and 49. Finite temperature predictions can also be found in Ref. [37].

3.3 Measurement of the fractional charge using noise

A difficulty of shot noise measurements in the FQH effect is that the extremely low shot noise has to be extracted from the background of relatively large amplifiers noise. Shot noise levels are extremely small both due to the smaller charge and the small available current. The latter is restricted by the fact that the FQH effect breaks down when the applied voltage is larger than the excitation gap. This excitation gap, in turn, depends crucially on the quality of the material in which the 2DEG resides. The state of the art technology currently yields samples with an excitation gap of the of the order of a few 100 μeV, leading to shot noise levels in the $10^{-29}A^2/Hz$ range.

These measurements have been performed in Saclay and in the Weizmann Institute [49][50]. A QPC is used in order to realize a local and controllable coupling between two $\nu = 1/3$ fractional edges to partially reflect the incoming current. The experiments are designed to have a best sensitivity for the weak coupling limit where Poissonian noise of the $e/3$ Laughlin quasiparticles is expected. In the experiment of Ref.[49], a cross correlation technique detects, at low

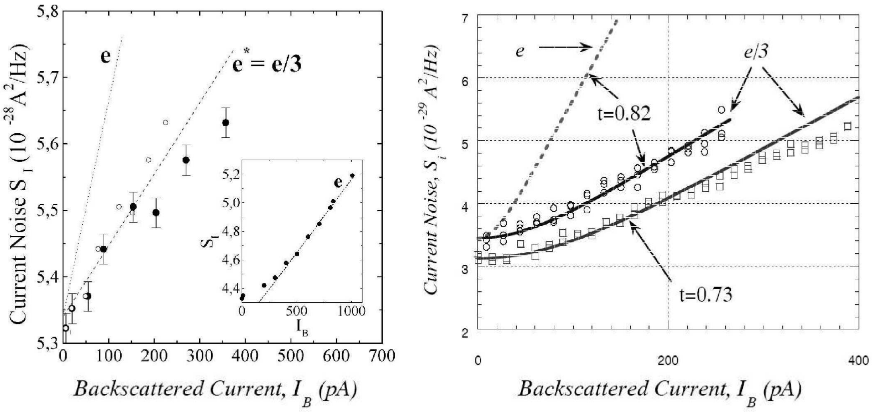

Figure 13: experimental Poissonian noise of the fractionally charged excitations in the FQHE, from Ref.[49] (left) and Ref.[50] (right).

frequency, the anticorrelated noise of the transmitted current I and the reflected current I_B, i.e. $S_{I,I_B} = \langle \Delta I \Delta I_B \rangle / \Delta f \simeq -2(e/3)I_B$, see Fig.12. In situ measurements of the Johnson-Nyquist noise versus temperature provide self-calibration of the current noise measured and are found consistent with independent calibration, so the shot noise is free of adjustable parameters. The magnetic field corresponds to a filling factor 2/3 in the bulk of the sample and a small region of filling factor 1/3 is created close to the QPC using the depletion effect of the gates. The size of the 1/3 region is estimated about 150 ϕ_0, sufficient to establish FQHE correlations. The advantage of doing this is that the coupling between edges occurs on a shorter scale and the controllable QPC potential is larger than the potential fluctuations inherent of sample fabrication. In the two samples measured, the combination of QPC and random potential lead to two dominant paths for backscattering. The coherent interference between paths gives rise to nearly perfect resonant tunneling peaks in the conductance. Careful measurements of the conductance resonance showed that tunneling was coherent. This was an important check for the quasiparticle charge measurement because this ruled out the possibility of noise suppression due to multiple uncorrelated hoping, similar to the 1/3 noise reduction factor in zero field diffusive conductors. Also the resonant conductance showed non-linear dependence on bias voltage consistent with Luttinger liquid model provided the filling factor of the bulk is used. The other group [50] used a high frequency technique in order to increase the signal bandwidth and measured the autocorrelation of the transmitted current. Here the magnetic field corresponded to a filling factor 1/3 everywhere in the sample. They found few non-linearities in the conductance, in contrast with the Luttinger liquid predictions, and this allowed them to define a bias voltage independent transmission.

In the Poissonian limit $I_B \ll I_0$, the two experiments give the same conclusion (see Fig.13) that near filling factor 1/3, shot noise is threefold suppressed. These experiments have given the most direct evidence that the current can be carried by quasiparticle with a fraction of e and that Laughlin conjecture was correct. In addition, the data showed a cross-over from thermal noise to shot noise when the applied voltage satisfies the inequality $eV/3 > 2k\theta$ (rather than $eV > 2k\theta$), indicating that the potential energy of the quasiparticles is threefold smaller as well as predicted in Eq.(11).This experiment has been now reproduced many time with different samples and measurement conditions in both laboratories.

Is it possible to go further and probe different fractional charges for less simple filling factor? Measurements close to $\nu = 2/5$ have given indications that the $e/5$ quasiparticles are the relevant excitations in this regime[51]. This last result has been analyzed in a model of non-interacting composite Fermions where Luttinger effects are neglected [52]. More recently, the same group has

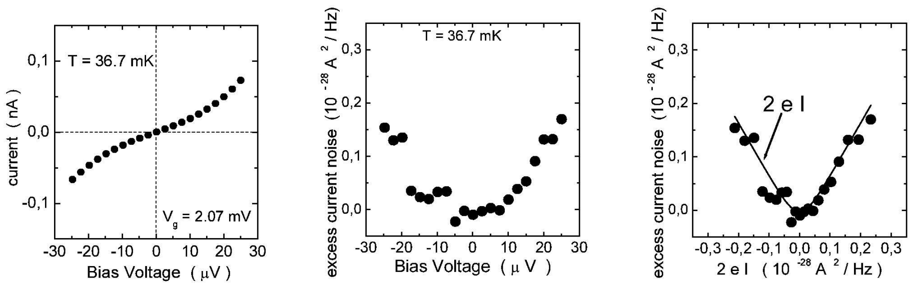

Figure 14: Although the QPC potential is very weak, at low temperature Luttinger liquid effects give rise to strong backscattering. The left figure shows the non-linear $I(V)$ curve, the center figure the shot noise versus voltage simultaneously measured, and the right figure the *linear* variation of the noise with current.

extended their measurement up to the filling factor 3/7 [53] and found a charge $e/7$. At very low temperature, they found an unexplained rapid increase of the effective charge. It would be useful to have a better theoretical understanding of the noise for Jain's filing factors to provide a basis for comparisons with experiments.

3.4 Cross-over from fractional to integer charge

When the strength of the artificial impurity potential is slightly increased and the energy of measurement is reduced, the strong backscattering regime sets in and one expects integer charges take over fractional charge. This has been measured in shot noise experiments [54],[55]. In ref.[54] very low temperature and pronounced Luttinger liquid effects are observed. In the same experiment it is then possible to observe both the charge $e/3$ for weak scattering and charge e at strong backscattering. Despite the strong linear I-V characteristics (due to Luttinger liquid effects), it is remarkable that the shot-noise increase linearly with current, as shown in Fig.14. In ref.[55] less non-linearities are observed as the electron temperature was higher but the strong backscattering regime was obtained by increasing the impurity strength.

4 Fractional occupation in the ground state

In the Laughlin wave function, electrons are spread uniformly with each elementary quantum state filled by $1/(2s+1)$ electrons on average.

A fractional filling is a necessary condition for the formation of fractionally charge excitations above the ground state. Indeed, the first excited state, a Laughlin wavefunction with a quasi-hole, is obtained by emptying a quantum state, i.e. by introducing a hole in the wavefunction whose area is that of one flux quantum. In a gedanken experiment, an infinitely small solenoid piercing the plane adiabatically increases the flux from zero to ϕ_0. In place of the fractionally filled quantum state a fractional charge $e/(2s+1)$ is left.

The fractional filling is also responsible for the fractional quantization of the Hall conductance. In a Corbino geometry (a ring of radius R and finite width $W \ll R$) it is possible to generate an azimuthal solenoidal electric field E_θ by a time varying flux $\Phi(t) = \phi_0 \frac{t}{\tau}$. If G_H is the Hall conductance, the radial current density is $j_r = G_H E_\theta$. For each time slice τ, the flux variation $\Delta\Phi = \phi_0$ radially shifts all states by one unit. As they are filled by a fraction of electron, a charge $\Delta Q = e/(2s+1)$ crosses the ring from the inner to the outer perimeter. The current is $I = j_r 2\pi R = G_H \frac{\phi_0}{\tau} = \frac{\Delta Q}{\tau}$ and the Hall conductance is $\frac{1}{2s+1}\frac{e^2}{h}$.

Equilibrium conductance measurements have allowed to accurately determine the fraction of charge filling the quantum states. Resonant tunneling experiments have been used to measure the charge ΔQ in response to the flux variation $\Delta\Phi = \phi_0$ through a well defined area for filling factor 1/3 and filling factor 1 [56]. Comparison between the integer and the fractional case showed that in the later case, the charge is accurately reduced by one third.

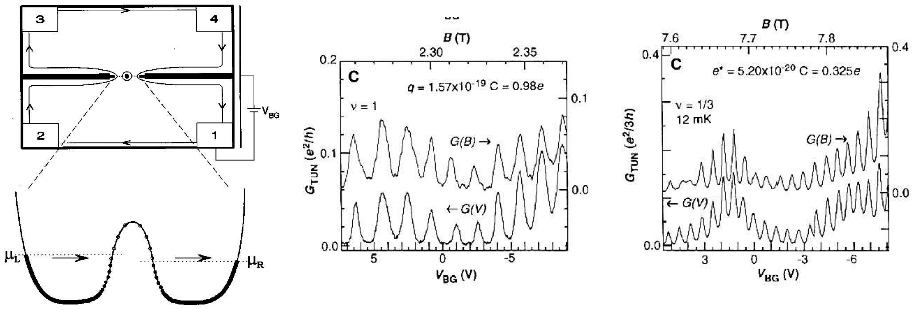

Figure 15: Resonant tunneling experiment to measure the fraction of charge associated with a quantum state at $\nu = 1/3$. Compared to the integer case, the period in gate voltage are increase by a factor 3 for 1/3 signaling a one third reduction of the charge.

In this experiment, a micron size disc fully depleted of electrons is realized inside a Hall bar by etching a 2D electron gas. The width of the Hall bar is locally reduced to a size larger but very close to the disc size, such that edge states running along the Hall bar can pass very close to the edge states circulating around the disc, see Fig.15. Gates, placed nearby the two points where the outer and inner edge channels meet, allow to control the tunnel coupling.

The disc free of electrons embedded in the 2DEG forms a so-called anti-dot (as opposed to quantum dots which are small disc of electrons). Because of the finite size of the perimeter and of the low temperature used, the available edge states do not appear as a continuum but are quantized. In a semiclassical picture, the radius r_k of the k^{th} edge state is given by $B\pi r_k^2 = k\phi_0$ and its energy is $U(r_k)$, where $U(r)$ is the radial potential which confines the electrons (the kinetic energy has been subtracted for presentation). Edge states with $U(r_k) < E_F$ are filled by one electron in the integer regime or by one third of electrons for $\nu = 1/3$. A new state can be filled by increasing the charge by ΔQ in the disc or by reducing the magnetic flux by ϕ_0. The filling of a new state is revealed by equilibrium measurement of the tunnel conductance between the upper and lower outer edge via the anti-dot. A resonant tunneling conductance peak is observed each time the edge state energy level align with the Fermi energy. The experiment shows that the same $\Delta\Phi = \phi_0$ separates two consecutive peaks for integer and fractional cases, while the backgate voltage variation ΔV separating two peaks is found one third smaller for the fractional case. Knowing the area of the antidot and the capacitance, the absolute variation of the charge ΔQ is found consistent with e and $e/3$ (estimation of the capacitance assumes strong statements about screening in the QHE regime, but the numbers are convincing). The results are shown in Fig.15.

5 Conclusion

The Fractional Quantum Hall effect is at present, the only system in condensed matter with fractional quantum numbers. Out of equilibrium or equilibrium tunneling experiments have been able to directly or indirectly probe fractionalization.

The Luttinger liquid properties which are revealed by power law variations of the tunnel conductance can not be understood without associating carriers with non integer charge. This fractionally charged carriers has been observed directly through the current noise associated with their tunneling across opposite boundaries of FQHE fluid. The evolution of the charge from a fraction in the weak backscattering limit at large voltage to an integer in the strong backscattering limit at low voltage is consistent with the Luttinger picture and with common intuition. At the root of the fractional excitations carrying the current is the fraction of charge in the ground state which fills the individual quantum states to form Laughlin's wavefunction. This fraction of charge has been measured by equilibrium resonant tunneling conductance measurements.

To complete this picture, localized fractional charges have been observed in a recent experiment using low temperature scanning probe imaging techniques [57]. The experiment is able to map the charge distribution in a macroscopic sample. The localized charges do not participate to transport (by definition) and are responsible for the finite width of the Hall plateau and exact quantization and is a key point in the understanding of the macroscopic QHE. Localized one third charge are found both for the 2/3 and the 1/3 FQHE regime.

Fascinating properties of the FQHE excitations are still to be observe. To cite a few: the fractional statistics which could be revealed by shot noise correlations techniques, the high frequency singularity in the shot noise at frequency $eV/(2s+1)h$, the fractional excitations in the non fully polarized spin regime. The recent progresses in mastering cold atoms suggest that the QHE could be observe in different systems with different statistics and interactions. New type of measurements may also be possible and extend our range of investigation of the Quantum Hall Effect.

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